OPTIMAL POWER CONTROL IN COGNITIVE MIMO SYSTEMS WITH LIMITED FEEDBACK

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ABSTRACT

In this paper, the problem of optimal power allocation in Cognitive Radio (CR) Multiple Input Multiple Output (MIMO) systems is treated. The focus is on providing limited feedback solutions aiming at maximizing the secondary system rate subject to a constraint on the average interference caused to primary communication. The limited feedback solutions are obtained by reducing the information available at secondary transmitter (STx) for the link between STx and the secondary receiver (SRx) as well as by limiting the level of available information at STx that corresponds to the link between the STx and the primary receiver PRx. Monte Carlo simulation results are given that allow to assess the performance achieved by the proposed algorithms.

Index Terms— Underlay Cognitive Radio, MIMO systems, power policy, ergodic rate maximization, average interference constraint.

1. INTRODUCTION

Cognitive Radio (CR) is considered an effective approach for coping with the spectrum scarcity problem in wireless communications systems. Among the several techniques that fall into the category of CR, underlay CR techniques have drawn considerable attention. The characteristic of such techniques is the fact that they allow the Secondary Transmitter (STx) to communicate with a Secondary Receiver (SRx) in the presence of a Primary Transmitter (PTx) communicating with a Primary Receiver (PRx), provided that the average or peak interference caused by STx transmission to PRx reception is below a predefined threshold. In this context, several underlay CR techniques aim at optimally allocating the STx transmit power in a manner such that some QoS metric, e.g. the average rate, of secondary communication is maximized subject to (s.t.) the constraint that the average or peak STx-PRx link interference is kept below a predefined threshold. For example, in [1] the optimal STx power policy is presented for systems operating under an average STx-PRx interference constraint, while in [2] several policies are derived based on

either average or peak power constraints regarding the STx transmit power and the STx-PRx link interference power.

Both these works, as well as most related works presented in the literature, are based on the so called "Z" channel model that assumes that there is no interference received by SRx, that corresponds to PTx transmission. More importantly, these works, are limited to the study of CR Single Input Single Output (SISO) systems. Motivated by the above, in this work we propose novel power policies for CR Multiple Input Multiple Output (MIMO) systems taking into account the average interference caused by secondary transmission to primary reception. The proposed policies are characterized by the fact that they require limited feedback sent from PRx and SRx to STx. More specifically the proposed policies, instead of requiring exact knowledge of the CR MIMO channel matrix $\mathbf{H}$, are based on the knowledge of only the eigenvalues of matrix $\mathbf{HH}^H$. Moreover, further feedback reduction schemes are derived, by introducing additional power policies that also assume only statistical CSI for the STx-PRx link.

2. SYSTEM MODEL

A CR MIMO system is considered that operates in the presence of a SISO primary link. We adopt the "Z" channel model where there is no interference caused by the primary transmission to secondary reception. In this case, the input output relation for the CR MIMO channel is given as

$$\mathbf{y}_s = \mathbf{H}\sqrt{P}\mathbf{x}_s + \mathbf{w}_s, \tag{1}$$

where $\mathbf{H}$ is the $M_R \times M_T$ channel matrix, M_R/M_T being the number of receive/transmit antennas for the cognitive MIMO system. Rayleigh fading is assumed where the elements of matrix $\mathbf{H}$ are i.i.d. zero mean complex Gaussian random variables with variance known to STx. Furthermore, $\mathbf{x}_s$ is the $M_T \times 1$ transmitted signal vector, assumed to be $\mathbf{x}_s \sim \mathcal{CN}(\mathbf{0}_{M_T}, \mathbf{I}_{M_T})$, where $\mathcal{CN}(\mathbf{m}, \mathbf{C})$ stands for the complex multivariate Gaussian distribution with mean $\mathbf{m}$ and covariance matrix $\mathbf{C}$. Matrix $\mathbf{I}_M$ stands for the $M \times M$ identity matrix and $\sim$ denotes equality in terms of distribution. In addition, P is the per antenna transmit power and $\mathbf{w}_s$ is the

Additive, White Gaussian Noise (AWGN) $M_R \times 1$ vector. For such a system model, the capacity of the secondary link, assuming STx has no knowledge of matrix $\mathbf{H}$, is given as [3]

$$C_s = \log_2 \det \left(\mathbf{I}_{M_R} + \frac{P}{N_0} \mathbf{H}\mathbf{H}^H \right) \qquad (2)$$

where $\det(\cdot)$ denotes the determinant and $\log_a(\cdot)$ the logarithm with base a. Using standard matrix properties, (2) can be written as

$$C_s = \sum_{i=1}^{M_R} \log_2 \left(1 + \frac{P}{N_0} l_i \right) \qquad (3)$$

where l_i's $i = 1, \ldots, M_R$ are the non negative eigenvalues of matrix $\mathbf{H}\mathbf{H}^H$.

We assume that the above CR system operates in the presence of a primary communication link, and causes interference to primary communication. As a result, the input-output relation for the primary communication link is expressed as

$$y_p = h_{pp}\sqrt{\mathcal{P}}x_p + \mathbf{h}_{sp}\sqrt{P}\mathbf{x}_s + w_p \qquad (4)$$

where h_{pp} is the PTx-PRx communication link, $\mathcal{P}$ is the constant transmit power used by the primary transmitter and $x_p \sim \mathcal{C}(0,1)$ the transmit symbol of the primary transmitter. On the other hand, $\mathbf{h}_{sp}$ is the $1 \times M_T$ channel vector between the secondary transmitter and the primary receiver and w_p the AWGN at the primary receiver side. By introducing the random variable $\eta = \|\mathbf{h}_{sp}\|^2$, we can write the average interference power caused by secondary transmission to primary reception as

$$\mathcal{U} = E\{\eta P\}. \qquad (5)$$

In the following analysis we will assume that the statistics, i.e. $\bar{\eta} = E\{\eta\}$ and the probability density function of η, are known to STx. Having defined the secondary system capacity as well as the average interference caused by the STx-PRx link, in the following sections we will present novel algorithms targeting on the maximization of the average achievable rate of the CR MIMO system. This maximization takes place s.t. a constraint on the average STx-PRx interference and a constraint on the maximum STx allowable peak power. Moreover the algorithms are designed in order to operate in the presence of limited feedback at STx. In the following sections, the novel algorithms are categorized according to the presumed PRx-STx feedback regarding the interference η and the level of SRx-STx feedback concerning the matrix $\mathbf{H}$.

3. ALGORITHMS BASED ON INSTANTANEOUS STX-PRX KNOWLEDGE

As a starting point, we consider that STx has exact instantaneous knowledge concerning the STx-PRx interference channel. Moreover, adopting a limited SRx-STx feedback scenario, we assume that STx has instantaneous knowledge of

only the eigenvalues l_i's $i = 1 \ldots, M_R$ of matrix $\mathbf{H}\mathbf{H}^H$. To the best of our knowledge this hypothesis has never been tested in the context of CR-MIMO links. The knowledge of such limited information does not suffice for applying the well known singular value decomposition (SVD) based approach [3] and the use of the waterfilling algorithm for capacity maximization. Nevertheless one can exploit the knowledge of eigenvalues in order to optimally allocate power across channel states. To this end, STx can apply power allocation across channel states by solving the following optimization problem

$$\text{maximize}: \quad E\left\{\sum_{i=1}^{M_R} \log_2 \left(1 + \frac{P}{N_0} l_i \right)\right\} \qquad (6)$$
$$\text{s.t. } \mathcal{U} = E\{\eta P\} \leq Q, \quad 0 \leq P \leq P_{\max}$$

or equivalently, after a change of the logarithm's base, the problem

$$\text{maximize}: \quad E\left\{\sum_{i=1}^{M_R} \ln \left(1 + \frac{P}{N_0} l_i \right)\right\} \qquad (7)$$
$$\text{s.t. } \mathcal{U} = E\{\eta P\} \leq Q, \quad 0 \leq P \leq P_{max}.$$

Notice that in this optimization problem, expectation is taken over the channel eigenvalues and the random variable η. Therefore in addition to knowledge of $\mathbf{H}$ and η, also knowledge of the second order statistics, i.e., the variance of the elements of the $\mathbf{H}$ channel matrix, and the statistics of η, i.e. $\bar{\eta}$ and its distribution, is required. Using this information, the power policy varies as a function of $\mathbf{H}$ and η while taking into account the average interference, positivity and peak power constraints. In what follows we will refer to the policy that solves the above optimization problem as the Eigenvalue Based Power Policy (EBPP). EBPP is presented in the following subsection.

3.1. Derivation of the EBPP

The EBPP can be found by first checking if the inequality

$$\bar{\eta}P_{max} \leq Q \qquad (8)$$

is satisfied. If this holds, then, the optimal power policy EBPP is readily expressed as

$$P_{EBPP} = P_{\max}. \qquad (9)$$

Otherwise, the optimization problem presented in (7) has the solution presented in the following theorem

Theorem 1. *If $\bar{\eta}P_{max} > Q$ the optimal power policy for optimization problem in* (7) *is given by*

$$P_{EBPP} = \begin{cases} 0 & \eta \geq \frac{\sum_{i=1}^{M_R} l_i}{\lambda N_0} \\ P_{max} & \eta \leq \sum_{i=1}^{M_R} \frac{l_i}{\lambda(N_0 + P_{max}l_i)} \\ \rho\left(\sum_{i=1}^{M_R} \frac{l_i}{N_0 + Pl_i} - \lambda\eta\right) & otherwise \end{cases}$$
$$(10)$$

where $\rho(f(P))$ stands for the root of equation $f(P) = 0$ with respect to P, and λ is selected such that $E\{\eta P_{EBPP}\} = Q$.

Proof. The optimization problem in (7) can be solved by noticing that due to concavity of the objective function and convexity of constraint functions, it is convex. Thus, the solution to this optimization problem can be found by applying KKT conditions. The details of the proof are not presented due to space limitations but can be found in [4]. □

Having derived the solution to the power alocation problem (7), in the following subsection we present a second power policy, namely the Maximum Eigenvalue Based Power Policy (MEBPP) that further reduces the need for feedback on the SRx-STx link since it requires only the knowledge of the maximum eigenvalue of $\mathbf{H}\mathbf{H}^H$.

3.2. Derivation of MEBPP

In this section we derive a second power policy that is based on the knowledge of only the maximum eigenvalue l_{max} of matrix $\mathbf{H}\mathbf{H}^H$. Based on this knowledge, STx can formulate the following optimization problem

$$\text{maximize:} \quad E\left\{\ln\left(1 + \frac{P}{N_0}l_{\max}\right)\right\} \tag{11}$$
$$\text{s.t.} \quad E\{\eta P\} \le Q, 0 \le P \le P_{max}.$$

In case that $\bar{\eta}P_{max} \le Q$ the solution to this optimization problem is simply $P_{MEBPP} = P_{\max}$. Otherwise, the solution to this problem is stated in the following theorem:

Theorem 2. *The solution to optimization problem* (11) *if* $\bar{\eta}P_{max} > Q$ *is given as*

$$P_{MEBPP} = \min\left\{\left[\frac{1}{\lambda\eta} - \frac{N_0}{l_{max}}\right]^+, P_{max}\right\} \tag{12}$$

where λ is the Lagrange multiplier, selected in order to satisfy the constraint $E\{\eta P_{MEBPP}\} = Q$.

Proof. The proof is derived following the steps of proof of Theorem 1 while setting $M_R = 1$ and $l_1 = l_{max}$. □

Having presented the two power policies, in the following section we will present further limited feedback power policies.

4. POWER POLICIES FOR FURTHER LIMITED CHANNEL FEEDBACK

In this section we present two new power policies that are based on a further limited feedback scenario. In more detail we examine the case that the secondary transmitter has only knowledge of the statistics of $\eta = ||\mathbf{h}_{sp}||^2$, i.e. knowledge of $\bar{\eta}$ and the distribution of η. In this case, the average constraint on the interference caused by STx to PRx is given as

$$\mathcal{V} = E\{\eta P\} = \bar{\eta}E\{P\}. \tag{13}$$

Using $\mathcal{V}$, in the following two subsections, we derive two new limited feedback power policies, the Interference statistics and Eigenvalue Based Power Policy (IEBPP) and the Interference statistics and Maximum Eigenvalue Based Power Policy (IMEBPP).

4.1. Derivation of IEBPP

The policy IEBPP is essentially the solution to the following optimization problem

$$\text{maximize:} \quad E\left\{\sum_{i=1}^{M_R}\ln\left(1 + \frac{P}{N_0}l_i\right)\right\} \tag{14}$$
$$\text{s.t.} \quad \mathcal{V} = \bar{\eta}E\{P\} \le Q, \ 0 \le P \le P_{max}.$$

In case that $\bar{\eta}P_{max} \le Q$ the solution to this problem is the trivial fixed policy $P = P_{\max}$. Otherwise, for the case $\bar{\eta}P_{max} > Q$, the derivation of the IEBPP is also based on the use of Lagrange Multipliers. As a result, IEBPP is defined as $P_{IEBPP} = P_{max}$ when $\bar{\eta}P_{max} \le Q$. Otherwise, P_{IEBPP} is defined as shown in the following theorem.

Theorem 3. *In case $\bar{\eta}P_{max} > Q$ The power policy is expressed as:*

$$P_{IEBPP} = \begin{cases} 0 & \bar{\eta} \ge \frac{\sum_{i=1}^{M_R} l_i}{\lambda N_0} \\ P_{max} & \bar{\eta} \le \sum_{i=1}^{M_R} \frac{l_i}{\lambda(N_0 + P_{max}l_i)} \\ \rho\left(\sum_{i=1}^{M_R} \frac{l_i}{N_0 + Pl_i} - \lambda\bar{\eta}\right) & \text{otherwise} \end{cases} \tag{15}$$

where λ is the Lagrange multiplier that is selected such as to satisfy the constraint $\bar{\eta}E\{P_{IEBPP}\} = Q$

Proof. The proof can be found in [4]. □

Similar to the case of EBPP, a limited feedback version of IEBPP, namely Interference statistics and Maximum Eigenvalue Based Power Policy (IMEBPP) can be constructed as shown in the following subsection.

4.2. Derivation of IMEBPP

The IMEBPP is derived as the solution to the following optimization problem

$$\text{maximize} \quad E\left\{\ln\left(1 + \frac{P}{N_0}l_{\max}\right)\right\} \tag{16}$$
$$\text{s.t.} \quad \bar{\eta}E\{P\} \le Q, \ 0 \le P \le P_{max}$$

where it is assumed that STx has knowledge of only the maximum eigenvalue $l_{\max}$ of matrix $\mathbf{H}$ and the statistics of the elements of matrix $\mathbf{H}$ mentioned earlier as well as knowledge of the statistics of η. As in the previous cases, the policy is derived by examining the two following cases: 1. $\bar{\eta}P_{max} \le Q$: In this case the optimal policy is the policy $P_{IMEBPP} = P_{\max}$. 2. $\bar{\eta}P_{max} > Q$: In this case the optimal policy is described in the following theorem.
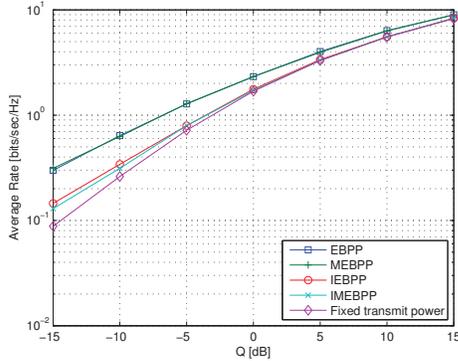
Fig. 1. Achievable performance of the presented policies in terms of average rate for a 2×2 system

Fig. 2. Achievable performance of the presented policies in terms of average rate for a 5×5 system.

Theorem 4. *The solution of problem* (16)*, in case that* $E\{\eta\} P_{max} > Q$ *leads to the power allocation* $P_{IMEBPP} = \min \left\{ \left[\frac{1}{\lambda \bar{\eta}} - \frac{N_0}{l_{\max}} \right]^+, P_{\max} \right\}$ *where* λ *is the Lagrange multiplier, selected in order to satisfy the constraint* $\bar{\eta} E\{P\} = Q$.

Proof. The proof can be found in [4]. □

5. COMPARISON OF THE DERIVED POLICIES

In this section we compare the several policies in terms of achievable rate. Specifically, in Fig. 1 we present the achievable rate for the several policies for a 2×2 MIMO system with $\bar{\eta} = M_T$ and $\mathbf{H}$ being a complex matrix with i.i.d entries being zero mean Gaussian and unit variance. P_{max} was set equal to $20dB$. We can observe that the policies EBPP and MEBPP achieve very similar results. Moreover, policies IEBPP and IMEBPP achieve also very similar results. Therefore, it can be deduced that knowledge of only the maximum eigenvalue of the channel matrix can lead to very similar performance as compared to the case that all eigenvalues are known. On the other hand we can see that there is a notable difference between the performance of policies EBPP and MEBPP as compared to the performance of IEBPP and IMEBPP. Finally, in our plot, we also include a curve corresponding to a fixed transmit policy with transmit power equal to $P = Q/\bar{\eta}$. Comparing this curve with curves corresponding to policies EBPP and MEBPP we can observe that even the knowledge of eigenvalue $l_{\max}$ and the exact knowledge of η can lead to a notable performance gain in terms of achievable rate. In Fig. 2 the performance of a 5×5 MIMO system is plotted for the proposed policies along with the performance of the fixed power policy with transmitted power $P = Q/\bar{\eta}$. Again we have selected $\bar{\eta} = M_T$ and $P_{max} = 20dB$. The performance gap between the several policies is reduced and Policies IEBPP, IMEBPP almost coincide, in terms of achievable rate with the fixed transmit policy. On the other hand, although the performance gap between Policies EBPP, MEBPP
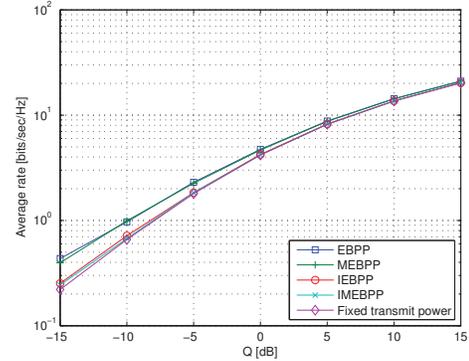
and the fixed transmit power is reduced, there still exists a clear performance gain for policies EBPP and MEBPP. Finally, by comparing results plotted in Figs. 1 and 2 it is evident that for all applied policies, the achievable gain increases as the number of transmit/receive antennas increases.

Acknowledgement

6. REFERENCES

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