# Energy Efficient Power Allocation for Carrier Aggregation Enabled Communications Systems

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***Abstract*—The Energy Efficiency (EE) of Carrier Aggregation (CA) enabled communications systems is studied. In more detail, the problem of the optimal power allocation for CA enabled systems exploiting multiple Parallel Frequency Bands (PFBs) is addressed based on an EE maximization criterion. Both the case of full Channel State Information at the Transmitter (CSIT) as well as the case of quantized CSIT are studied and novel EE-optimal power allocation algorithms are derived. The derived algorithms are used in order to investigate the relation between the system's EE and the system design parameters, e.g. the number of PFBs, the number of feedback bits and the fading conditions. Focusing on the problem of optimal feedback allocation, it is shown by means of simulations that the optimal allocation of feedback to the PFBs is related to the fading conditions. Moreover, focusing on the problem of the selection of the number of PFBs, it becomes evident that the optimal selection of the number of PFBs is strongly related to the number of available feedback bits. Thereby, the interplay between EE-optimal feedback and power allocation design and system parameters is explored.**

***Keywords*—*Energy Efficiency, Optimal Power Allocation, Carrier Aggregation, Quantized Feedback.***

## I. Introduction

The design of optimal power allocation schemes in the presence of Channel State Information at the Transmitter (CSIT) allows the exploitation of the theoretical performance limits of wireless communications systems operating in fading environments. Thus, in the open technical literature, several power allocation schemes for wireless communication systems have been presented. In more detail, in [1], in the presence of perfect CSIT, starting from the problem of the maximization of the ergodic capacity subject to an average power constraint, the well-known waterfilling policy is derived for Single Input Single Output (SISO) systems. In [2]–[5], assuming again perfect CSIT, ergodic rate maximization power policies for Cognitive Radio (CR) systems are derived, taking into account average transmit power and/or average secondary-to-primary interference constraints. While all previous works focus on the ergodic rate maximization of wireless communications systems, yet another performance criterion that has recently emerged for the design of power allocation policies is the *Energy Efficiency* (EE). This performance analysis criterion has been investigated in [6], where the power policy for maximizing the EE of SISO wireless communications systems is derived taking into account the delay-outage constraints of the transmitted data as well as the non-zero circuit power consumption. The derived power policy assumes perfect CSIT.

Recently, Carrier Aggregation (CA) has appeared as a novel system design technique in wireless communications systems. Based on ideas similar to CR, CA allows for increasing the achievable rate of communications systems by using parallel communications channels, and opportunistically exploiting unused carriers. The potential benefits offered by CA have led to its inclusion in several Mobile Communications standards, e.g. Long Term Evolution - Advanced (LTE-Advanced) [7], [8]. In [8], the EE for the uplink of CA enabled systems is investigated assuming a fixed transmit power scheme. A similar analysis for the downlink of CA enabled systems is presented in [9]. While [8] and [9] focus on homogeneous networks, in the literature, the concept of CA is also being studied for Heterogeneous Networks (HetNets) [10], [11]. Nevertheless, regardless of the context, the problem of EE-optimal power allocation for CA enabled systems is still an open issue.

An interesting problem that appears when attempting to design EE-optimal power policies for CA enabled systems exploiting multiple Parallel Frequency Bands (PFBs), is the increase in feedback, i.e. CSI, for the several PFBs that needs to be sent to the transmitter [12]. In the presence of rate limited feedback links, this raises the demand to also account for the effects of, potentially coarse, feedback quantization on the optimal power allocation design. The effects of quantized feedback on wireless communication systems have been studied extensively. In more detail, in [13] several system designs exploiting limited, digital, feedback in wireless communications systems are proposed. Single and Multi-antenna systems are examined within both single and multi-user frameworks. In [14] the influence of limited feedback on the capacity of MIMO systems is investigated. In [15] the effects of quantized CSIT on the performance of CR systems is investigated. Moreover, in [16] the problem of the per user optimal feedback and power allocation for the Multiple Input Single Output (MISO) downlink of multiuser systems is studied such as to maximize the EE of the system. Finally, in [17] the problem of rate optimal power allocation in multiuser environments with quantized CSIT is considered. The focus of all these works though is on systems employing a single frequency band. To the best of our knowledge, no results concerning the problem of optimal power allocation for CA enabled systems in the presence of quantized CSIT can be found.

Motivated by the above, in this work we investigate the problem of EE-optimal power allocation in CA enabled communication systems. Specifically, it is assumed that the

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multiple PFBs become available to the user of interest in a probabilistic manner. For such a system model, initially, the problem of EE-optimal power allocation is formulated and solved assuming perfect CSIT, while in the sequel the case of imperfect, quantized CSIT is considered. In more detail, the contribution of our paper is summarized as follows: a) The EE-optimal power allocation policy for CA enabled systems in the presence of perfect CSIT is derived in closed form. b) The case of quantized CSIT is also investigated and a technique is developed such as to find the EE-optimal transmit power allocation policy for this case. c) Based on the derived policy, focusing on limited feedback systems constrained by a total number of feedback bits, based again on an EE maximization criterion, the dependency of the optimal allocation of quantization bits upon the fading statistics is investigated by means of simulations. Moreover, the problem of the optimal selection of the number of PFBs is also investigated.

The paper is structured as follows. In Section II the considered system model is presented. In Section III the EE-optimal power allocation policy in the presence of perfect CSIT is derived, while in Section IV the optimal power policy for the case of quantized CSIT is presented. Moreover, in Section V numerical results obtained by applying the proposed algorithms are shown. With the aid of the obtained results, the relation of the system parameters, i.e. number of PFBs, number of total available feedback bits, fading conditions and probability of availability of the several PFBs with the optimal feedback allocation and selection of the number of PFBs is investigated. Finally, in Section VI some conclusions are given.

*Notation:* Bold letters are used to denote vectors. We use $E_1(\cdot)$ to denote the exponential integral function [18, eq. (5.1.1)]. Operator $[x]^+$ stands for $\max\{x,0\}$, and $|\cdot|_1$ stands for the $l-1$ norm. The Probability Distribution Function (PDF) of a random vector $\mathbf{x}$ is denoted a s $p_{\mathbf{x}}(x)$. Notation $\mathbb{E}\{\cdot\}$ is used to denote the expectation of a random variable and notation $\mathbb{E}_{\mathbf{x}}\{f(\mathbf{x})\}$ is used to denote the expectation of the function $f(\mathbf{x})$ of a random vector $\mathbf{x}$. The probability of an event $\mathcal{A}$ is denoted as $\Pr(\mathcal{A})$.

## II. System Model

A Single Input Single Output (SISO) communications system is considered where communication is achieved using several PFBs by means of CA. That is, the user of interest employs a Frequency Band (FB) $\mathcal{B}_0$ assigned to it for its transmission as well as, by means of CA, the FBs out of the set $\{\mathcal{B}_1,\ldots,\mathcal{B}_M\}$ that are not employed by any other user of the system. Within a HetNet CA scheme, such a scenario is encountered for example in the case of a two-tier network consisting of macrocells and small-cells used in order to assist communication as in [19]. In such a case, band $\mathcal{B}_0$ may correspond to the FB allocated to the user of interest by the marcocell and FBs $\mathcal{B}_1,\ldots,\mathcal{B}_M$, may correspond to the carriers allocated to the same user by nearby small-cells. On the other hand in case of a homogeneous network, such a scenario may exist when a user exploits, in a CR fashion, resources allocated to other users or cells of the network.

In our analysis we assume that the event that band $\mathcal{B}_{m_1}$ is available for transmission by the user of interest, is independent of the event that band $\mathcal{B}_{m_2}$ is available for transmission for $m_1 \neq m_2$. Moreover, we assume that the probability that frequency band $\mathcal{B}_m$ is free for transmission is the same for all $m=1,\ldots,M$, and equal to $\pi_1$. Having defined the system model, one can express the instantaneous achievable rate of communication through band $\mathcal{B}_0$, in $nats/sec$, as:

$$R_0 = B_0 \log\left(1+\frac{g_0 P_0}{N_0 B_0}\right), \tag{1}$$

where $B_0$ is the bandwidth of $\mathcal{B}_0$, $g_0$ stands for the flat fading coefficient on band $\mathcal{B}_0$, $P_0$ is the transmit power allocated instantaneously to the transmission taking place at band $\mathcal{B}_0$ and $N_0$ is the noise spectral density. On the other hand, the instantaneous achievable rate on band $\mathcal{B}_m$, $m=1,\ldots,M$, is given as:

$$R_m = B_m \log\left(1+\frac{I_m g_m P_m}{N_0 B_m}\right), \tag{2}$$

where $B_m$ is the bandwidth of band $\mathcal{B}_m$, $g_m$ is the flat fading coefficient on band $\mathcal{B}_m$, $P_m$ is the transmit power allocated instantaneously for the transmission taking place through band $\mathcal{B}_m$, and $I_m$ is a binary Random Variable (RV) modeling the availability of band $\mathcal{B}_m$. That is, it holds that $I_m=1$, if $\mathcal{B}_m$ is available and $I_m=0$ if $\mathcal{B}_m$ is already used by another user. Clearly, it holds that $\Pr(I_m=1)=\pi_1$.

Assuming the existence of instantaneous knowledge of $\mathbf{g}=[g_0,\ldots g_M]$ and $\mathbf{I}=[I_1,\ldots,I_M]$ at the Transmitter (Tx) of the communication system, we consider the case that Tx adapts the transmit power levels $P_0,\ldots P_M$, as a function of this information, i.e., by employing power levels of the form $P_0(\mathbf{g},\mathbf{I}),\ldots,P_M(\mathbf{g},\mathbf{I})$ [1]. Thus, one can write the achievable average communication rate, measured in $nats/sec$, for the investigated communication system as:

$$C = \mathbb{E}_{\mathbf{g}}\left\{\sum_{\mathbf{I}\in\mathcal{I}}\sum_{m=0}^{M}\Pr(\mathbf{I})\, B_m \log\left(1+\frac{I_m g_m P_m(\mathbf{g},\mathbf{I})}{N_0 B_m}\right)\right\}, \tag{3}$$

where the outer summation in (3) takes place over the set $\mathcal{I}$ of all realizations for vector $\mathbf{I}$. Moreover, $\Pr(\mathbf{I})$, i.e. the probability of occurrence of vector $\mathbf{I}$, in (3) is expressed as:

$$\Pr(\mathbf{I}) = \pi_1^{|\mathbf{I}|_1}(1-\pi_1)^{M-|\mathbf{I}|_1}. \tag{4}$$

Hence, one can express the EE of the investigated system, measured in nats per Joule as [6]:

$$E_{eff} = \frac{C}{P_c+\bar{P}}, \text{where } \bar{P} = \mathbb{E}_{\mathbf{g}}\left\{\sum_{\mathbf{I}\in\mathcal{I}}\sum_{m=0}^{M}\Pr(\mathbf{I})\, P_m(\mathbf{g},\mathbf{I})\right\}, \tag{5}$$

where $P_c$ is a fixed term corresponding to the power consumption of the Tx electronic circuits [6]. We are interested in the design of power policies $P_m(\mathbf{g},\mathbf{I}), m=0,\ldots,M, I_m\neq 0$, such as to maximize the system's EE. This problem is formulated and solved in the following section. Following that, we revise the system model such as to encompass for the effects of quantized CSIT. For this revised system model, we reformulate and solve the problem of optimal power allocation.

[1] Clearly it holds that $P_m(\mathbf{g},\mathbf{I})=0$ if $I_m=0$, for $m=1,\ldots,M$, since in this case band $\mathcal{B}_m$ is not available for transmission.

## III. Energy Efficient Power Allocation with Perfect Feedback

The problem of the optimal design of the power allocation policies $P_m(\mathbf{g},\mathbf{I}), m = 0,\dots,M$, provided that (p.t.) $I_m \neq 0,\ m \geq 1$, can be mathematically expressed as:

$$\begin{aligned}&\underset{P_m(\mathbf{g},\mathbf{I}),\ m=0,\dots,M,\ \text{p.t. } I_m\neq 0, m\geq 1}{\text{maximize:}} \quad E_{eff}\\ &\text{subject to: } P_m(\mathbf{g},\mathbf{I}) \geq 0,\ m=0,\dots,M, \text{p.t. } I_m \neq 0, m\geq 1.\end{aligned} \tag{6}$$

Based on the fact that function $f(x) = \log(1+\alpha x), \alpha \geq 0$ is concave, one can see that the cost function $E_{eff}$ in (6) is the ratio of a concave function, i.e. the capacity $C$, and an affine function. Hence, one can solve problem (6) by introducing the auxiliary variable $u$, and applying fractional programming principles [6], [20] such as to transform problem (6) to the following optimization problem:

$$\begin{aligned}&\underset{P_m(\mathbf{g},\mathbf{I}),\ m=0,\dots,M,\ \text{p.t. } I_m\neq 0,\ m\geq 1}{\text{maximize:}} \quad uC\\ &\text{subject to:} \quad u\left(P_c+\bar{P}\right)=1,\\ &P_m(\mathbf{g},\mathbf{I}) \geq 0,\ \ m=0,\dots,M,\ \text{p.t. } I_m\neq 0,\ m\geq 1.\end{aligned} \tag{7}$$

The solution to optimization problem (7) is given in the following theorem.

*Theorem 1:* The optimal power allocation policies such as to maximize (7) are given as

$$\begin{aligned}&P_m(\mathbf{g},\mathbf{I}) = \left[\frac{B_m}{\lambda} - \frac{N_0 B_m}{g_m}\right]^+, m=0,\dots,M,\\ &\text{provided that} \quad I_m \neq 0, \ \text{if} \ \ m\geq 1\end{aligned} \tag{8}$$

where $\lambda$ is a Lagrange multiplier selected such as to satisfy the constraint

$$C = \lambda\left(P_c+\bar{P}\right). \tag{9}$$

*Proof:* One can easily show that optimization problem (7) is convex with respect to optimization variables $P_m(\mathbf{g},\mathbf{I})$, $m=0,\dots,M$, where, $I_m \neq 0$ for $m\geq 1$ and $u$. Hence it can be solved by applying the Karush-Kuhn-Tucker (KKT) conditions [21]. Specifically, by introducing the Lagrange Multiplier $\lambda$ for constraint (9) as well as Lagrange Multipliers for the non-negative transmit power constraints and differentiating the resulting Lagrangian function with respect to optimization variables $P_m(\mathbf{g},\mathbf{I}), m=1,\dots,M$, and $u$, it is shown that the optimal power policies are found as the solution to the following set of equations[2]

$$\begin{aligned}&P_m(\mathbf{g},\mathbf{I}) = \frac{B_m}{\lambda-\mu_m(\mathbf{g},\mathbf{I})} - \frac{N_0 B_m}{g_m}, m=0,\dots,M,\\ &\mu_m(\mathbf{g},\mathbf{I})P_m(\mathbf{g},\mathbf{I})=0, P_m(\mathbf{g},\mathbf{I})\geq 0, \mu_m(\mathbf{g},\mathbf{I})\geq 0,\\ &C=\lambda\left(P_c+\bar{P}\right), u\left(P_c+\bar{P}\right)=1\end{aligned} \tag{10}$$

where $\mu_m(\mathbf{g},\mathbf{I})$ is the Lagrange Multiplier corresponding to the constraint $P_m(\mathbf{g},\mathbf{I}) \geq 0$. Thus, after some simple manipulations, it is easy to show that the optimal power allocation policies are given as in (8). Note that based on the derived solution for the optimal power allocation, the determination of the optimal value for variable $u$ in (6) is not required. Finally, it should be mentioned that the value of Lagrange multiplier $\lambda$ can be determined by applying a bisection algorithm to find the root of (9) with respect to $\lambda$. ■

[2]Provided again that $I_m \neq 0$ if $m \geq 1$.

Having solved the problem of optimal power allocation for the case of perfect CSIT, in the following section, we investigate the problem of optimal power allocation for the case of quantized CSIT.

## IV. Energy Efficient Power Allocation with Quantized Feedback

Let us consider the case where Tx, along with knowledge of $\mathbf{I}$, has quantized knowledge concerning the values of fading coefficients $g_m, m=0,\dots M$. Let $N_{Q,m}, m=0,1,\dots,M$, denote the number of quantization levels used for the quantization of $g_m$. In such a case, the information available at Tx is essentially the information that event $\epsilon_{m,i_m}$, defined as $\epsilon_{m,i_m} : Q_{m,i_m} \leq g_m \leq Q_{m,i_m+1}, m=0,\dots M$, has occurred, for some $i_m \in \{0,\dots N_{Q,m}-1\}$ where $Q_{m,i_m}, i_m = 0,1,\dots,N_{Q,m}$, are the quantization thresholds for $g_m$, with $Q_{m,0}=0$ and $Q_{m,N_{Q,m}}=\infty$. By defining the vector $\mathbf{e} = [\epsilon_{0,i_0},\dots \epsilon_{M,i_M}]$ and the set $\mathcal{E}$ of all possible realizations for vector $\mathbf{e}$, we are then interested in finding the optimal power policies $\hat{P}_m(\mathbf{e},\mathbf{I}), I_m \neq 0$[3] such as to maximize the EE of the communications system. To this end, in the following subsection we first express the EE for the case of quantized CSIT in closed form.

### A. Energy Efficiency Analysis with Quantized Feedback

In the presence of quantized CSIT, one can express the achievable rate of the investigated communication system as:

$$\hat{C} = \sum_{\mathbf{I}\in\mathcal{I}} \Pr(\mathbf{I}) \sum_{\mathbf{e}\in\mathcal{E}} \hat{R}\left(\hat{P}_0(\mathbf{e},\mathbf{I}),\dots,\hat{P}_M(\mathbf{e},\mathbf{I}),\mathbf{e},\mathbf{I}\right), \tag{11}$$

where $\hat{R}(\cdot)$ is defined as in (12a). Restricting ourselves to the case of independent Rayleigh fading, such that one can write $p_{\mathbf{g}}(\mathbf{g}) = \prod_{m=0}^{M} p_{g_m}(g_m)$, with

$$p_{g_m}(g) = \frac{1}{\bar{g}_m}\exp\left(-\frac{g}{\bar{g}_m}\right), \text{where} \quad \bar{g}_m = \mathbb{E}\{g_m\}, \tag{13}$$

we can express $\hat{R}(\cdot)$ as in (12b) where we have used property [22, eq. 4.337.1].

On the other hand, the average consumed power, this time is expressed as:

$$\hat{P} = \sum_{\mathbf{I}\in\mathcal{I}} \Pr(\mathbf{I}) \sum_{\mathbf{e}\in\mathcal{E}} \Pr(\mathbf{e}) \sum_{m=0}^{M} \hat{P}_m(\mathbf{e},\mathbf{I}). \tag{14}$$

Hence, the EE in case of Quantized CSIT is expressed as:

$$\hat{E}_{eff} = \frac{\hat{C}}{P_c+\hat{P}}. \tag{15}$$

In the following subsection we address the problem of the design of the optimal power allocation policies $\hat{P}_m(\mathbf{e},\mathbf{I}), m = 0,\dots,M$, such as to maximize the EE in (15).

[3]Clearly, similarly to the case of perfect CSIT, $\hat{P}_m(\mathbf{e},\mathbf{I}) = 0$ if $I_m = 0$.

$$\hat{R}\left(\hat{P}_0(\mathbf{e},\mathbf{I}),\ldots,\hat{P}_M(\mathbf{e},\mathbf{I}),\mathbf{e},\mathbf{I}\right)=\sum_{m=0}^{M}\int_{Q_{M,i_M}}^{Q_{M,i_M+1}}\cdots\int_{Q_{0,i_0}}^{Q_{0,i_0+1}} B_m\log\left(1+\frac{I_m g_m \hat{P}_m(\mathbf{e},\mathbf{I})}{N_0 B_m}\right)p_{\mathbf{g}}(\mathbf{g})\,d_{g_0}\cdots d_{g_M}. \tag{12a}$$

$$\text{For independent Rayleigh fading: } \hat{R}\left(\hat{P}_0(\mathbf{e},\mathbf{I}),\ldots,\hat{P}_M(\mathbf{e},\mathbf{I}),\mathbf{e},\mathbf{I}\right)=\Pr(\mathbf{e})\sum_{m=0}^{M}\hat{C}_{\epsilon_{m,i_m},I_m}, \tag{12b}$$

$$\text{where}\quad \hat{C}_{\epsilon_{m,i_m},I_m}=\frac{B_m}{\Pr(\epsilon_{m,i_m})}\left(U\left(Q_{m,i_m},\frac{I_m\hat{P}_m(\mathbf{e},\mathbf{I})}{N_0B_m},\bar{g}_m\right)-U\left(Q_{m,i_m+1},\frac{I_m\hat{P}_m(\mathbf{e},\mathbf{I})}{N_0B_m},\bar{g}_m\right)\right) \tag{12c}$$

$$\text{with}\quad U(\alpha,\beta,\gamma)=\exp\left(-\frac{\alpha}{\gamma}\right)\left(\log(1+\alpha\beta)+\exp\left(-\frac{1+\alpha\beta}{\beta\gamma}\right)E_1\left(\frac{1+\alpha\beta}{\beta\gamma}\right)\right). \tag{12d}$$

### B. Optimal Power Allocation with Quantized Feedback

The problem of the optimal power allocation in the presence of quantized CSIT is formulated as:

$$\begin{aligned}&\underset{\hat{P}_m(\mathbf{e},\mathbf{I}),m=0,\ldots,M,\text{ p.t. } I_m\neq 0, m\geq 1}{\text{maximize:}}\quad \hat{E}_{eff}\\ &\text{subject to: } \hat{P}_m(\mathbf{e},\mathbf{I})\geq 0,\ m\geq 0,\ \text{p.t. } I_m\neq 0,\ m\geq 1.\end{aligned} \tag{16}$$

Introducing again an auxiliary variable $u$ and applying fractional programming tools, we can transform this problem to the following problem:

$$\begin{aligned}&\underset{\hat{P}_m(\mathbf{e},\mathbf{I}),\text{ p.t. } I_m\neq 0,m\geq 1}{\text{maximize:}}\quad u\hat{C}\\ &\text{subject to: } \quad u\left(P_c+\hat{P}\right)=1,\\ &\hat{P}_m(\mathbf{e},\mathbf{I})\geq 0, m=0,\ldots,M,\ \text{p.t. } I_m\neq 0, m\geq 1.\end{aligned} \tag{17}$$

The solution to this problem is given by Theorem 2.

*Theorem 2:* The optimal power allocation policies $\hat{P}_m(\mathbf{e},I), m=0,\ldots,M,$ p.t. $I_m\neq 0,$ $m\geq 1$ for solving optimization problem (17) are given as the root of the equations[4]:

$$\frac{\partial\hat{R}\left(\hat{P}_0(\mathbf{e},\mathbf{I}),\ldots,\hat{P}_M(\mathbf{e},\mathbf{I}),\mathbf{e},\mathbf{I}\right)}{\partial\hat{P}_m(\mathbf{e},\mathbf{I})}-\lambda\Pr(\mathbf{e})=0 \tag{18}$$

where the Lagrange Multiplier $\lambda$ is selected such that the condition $\hat{C}=\lambda\left(P_c+\hat{P}\right)$ is satisfied.

*Proof:* It is easy to show that similarly to the case of perfect CSIT at Tx, optimization problem (17) is convex. As a result, it can be solved by applying KKT conditions, i.e. by constructing the Lagrangian function and differentiating with respect to optimization variables $\hat{P}_m(\mathbf{e},\mathbf{I})$, $m=0,\ldots,M$, and $u$. It is then easy to show that power allocation policies are given by numerically solving equation (18). Similar to the case of perfect CSIT, the determination of the optimal value for the auxiliary variable $u$ is not required for calculating the optimal power policy. Moreover, the Lagrange multiplier $\lambda$ can be caclulated using a bisection algorithm to solve the equation $\hat{C}=\lambda\left(P_c+\hat{P}\right)$. ■

[4]These equations need to be solved again only for $I_m\neq 0$. If $I_m=0$ obviously it holds that $\hat{P}_m(\mathbf{e},\mathbf{I})=0$.

Note that conditions (18) can be rewritten in a simplified form, by noticing that:

$$\frac{\partial\hat{R}\left(\hat{P}_0(\mathbf{e},\mathbf{I}),\ldots,\hat{P}_M(\mathbf{e},\mathbf{I}),\mathbf{e},\mathbf{I}\right)}{\partial P_m(\mathbf{e},\mathbf{I})}=\Pr(\mathbf{e})\frac{\partial\hat{C}_{\epsilon_{m,i_m},I_m}}{\partial P_m(\mathbf{e},\mathbf{I})}, \tag{19}$$

and employing [18, eq. (5.1.27] for the derivative of the exponential integral $E_1(\cdot)$. Moreover, another interesting remark comes from the fact that based on (18) and (19), power allocation $\hat{P}_m(\mathbf{e},\mathbf{I})$ depends solely on the value of the Lagrange Multiplier $\lambda$ that is determined by the statistics of the fading channel, and on the instantaneous channel realization $g_m$, i.e. one can rewrite $\hat{P}_m(\mathbf{e},\mathbf{I})$ as $\hat{P}_m(\epsilon_{m,i_m},I_m)$. This fact allows for simplifying the procedure for calculating the Lagrange Multiplier $\lambda$ that is the solution to the equation $\hat{C}=\lambda\left(P_c+\hat{P}\right)$. In more detail, by substituting (12) into (11), it is easy to show that due to the form of the optimal power policy for FBs $\mathcal{B}_m, m=0,\ldots M$, by rewriting $\Pr(\mathbf{e})$ as $\Pr(\mathbf{e})=\Pr(\tilde{\mathbf{e}}_m)\Pr(\epsilon_{m,i_m})$, and exchanging the order of summation over all possible realizations of vector $\mathbf{e}$ and over $m=1,\ldots,M$ in the definition of $\hat{R}(\cdot)$, it is easy to rewrite $\hat{C}$ in the following form:

$$\begin{aligned}\hat{C}=&\sum_{\mathbf{I}\in\mathcal{I}}\Pr(\mathbf{I})\sum_{m=0}^{M}\sum_{\tilde{\mathbf{e}}_m\in\tilde{\mathcal{E}}_m}\Pr(\tilde{\mathbf{e}}_{\mathbf{m}})\,\mathbb{E}_{\epsilon_{m,i_m}}\left\{\hat{C}_{\epsilon_{m,i_m},I_m}\right\}=\\ &\sum_{\mathbf{I}\in\mathcal{I}}\Pr(\mathbf{I})\sum_{m=0}^{M}\mathbb{E}_{\epsilon_{m,i_m}}\left\{\hat{C}_{\epsilon_{m,i_m},I_m}\right\},\end{aligned} \tag{20}$$

where $\tilde{\mathcal{E}}_m$ denotes the set of all possible realizations for vector $\tilde{\mathbf{e}}_m$. In a similar manner, it is easy to show that:

$$\hat{P}=\sum_{\mathbf{I}\in\mathcal{I}}\Pr(\mathbf{I})\sum_{m=0}^{M}\mathbb{E}_{\epsilon_{m,i_m}}\left\{\hat{P}_m(\epsilon_{m,i_m},I_m)\right\}. \tag{21}$$

## V. Numerical results

In this section, we present the achievable performance of the proposed power allocation policies for different system setups. In Fig. 1, we present the achievable EE for a system with two PFBs, i.e. bands $\mathcal{B}_0$ and $\mathcal{B}_1$, as a function of the total number of bits used for the quantization of information $g_0$ and $g_1$ that is fed back to Tx. The presented results have been obtained by selecting the optimal feedback allocation that maximizes the achievable EE. In more detail, if $N_{b,0}^*$ and $N_{b,1}^*$ represent the number of bits used for the quantization of $g_0$ and $g_1$ respectively, leading to quantization schemes

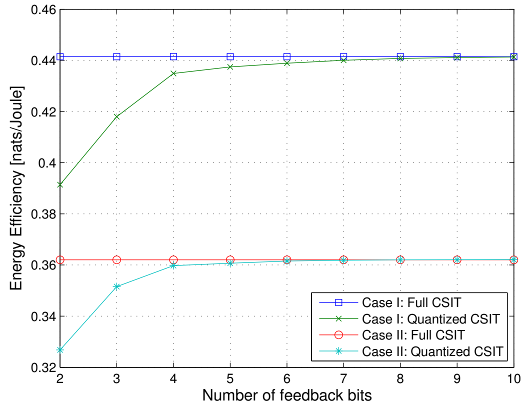


Fig. 1. Optimal achievable energy efficiency as a function of the number of quantization bits

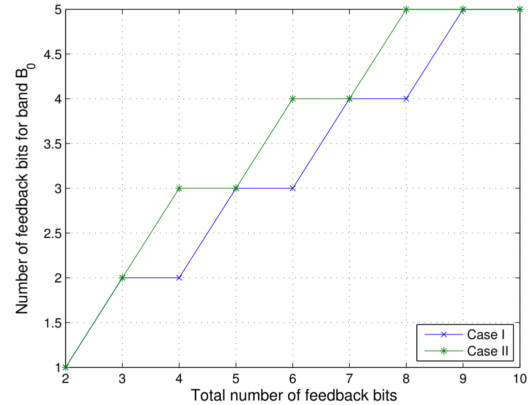


Fig. 2. Number of bits allocated to the quantization of band $\mathcal{B}_0$ for the two examined cases.

with $N_{Q,0}^* = 2^{N_{b,0}^*}$ and $N_{Q,1}^* = 2^{N_{b,1}^*}$ levels for $g_0$ and $g_1$ respectively, and $N_b$ is the total number of bits used for feedback at each transmission block, we obtain $N_{b,0}^*$ and $N_{b,1}^*$ by solving the optimization problem:

$$\begin{aligned} &\underset{N_{b,0},N_{b,1}}{\text{maximize:}} \;\; \hat{E}_{eff}\left(N_{b,0}, N_{b,1}\right) \\ &\text{subject to:} \;\; N_{b,0} + N_{b,1} = N_b, \;\; N_{b,0}, \;\; N_{b,1} \in \mathbb{N}^+, \end{aligned} \tag{22}$$

where $\hat{E}_{eff}\left(N_{b,0}, N_{b,1}\right)$ stands for the achievable EE, for the case that the power policies of Theorem 2 are applied while setting $N_{Q,0} = 2^{N_{b,0}}$ and $N_{Q,1} = 2^{N_{b,1}}$ in the process of designing the quantization codebook. Clearly, the value of $\hat{E}_{eff}\left(N_{b,0}, N_{b,1}\right)$ depends also on the procedure followed for the design of the codebooks for $g_0$ and $g_1$. In this work, we adopt the use of Lloyd-Max quantizers. Moreover, in Fig. 1, we also present the achievable EE for the case of full CSIT. The presented results have been obtained by considering the two following different cases for the relation between $\bar{g}_0$ and $\bar{g}_1$: 1) In Case I we have considered a scenario where $\bar{g}_0 = \bar{g}_1 = 1$. 2) In Case II we have considered a scenario where $\bar{g}_0 = 1$ and $\bar{g}_1 = 0.2$. For both cases, for the two bands we have assumed that $B_0 = B_1 = 1Hz$, and we have set the noise density $N_0$ to be equal to 1. Moreover, the probability that the band $\mathcal{B}_1$ is available to the user under investigation, was set to be equal to $\pi_1 = 0.6$. The circuits related power consumption $P_c$ was set to be equal to $P_c = N_0 = 1$ as in [6]. As it can be observed in Fig. 1, as the number of quantization bits $N_b$ increases, the achievable EE also increases and converges to the achievable EE for the case of full CSIT. Moreover, the fact that in Case I, the average fading power on band $\mathcal{B}_1$ is higher than in Case II leads to higher EE.

Fig. 2 depicts the optimal number of bits $N_{b,0}^*$ allocated to the quantization of band $\mathcal{B}_0$ that leads to the EE maximization, as it is found by solving problem (22), for different values of $N_b$. The results shown in Fig. 2 clearly indicate that fading statistics on the available FBs influence the optimal feedback allocation. As it can be seen, the fact that fading effects are more severe on band $\mathcal{B}_1$ in Case II, leads to allocating fewer bits to this band in Case II rather than in Case I. As a result, the number of bits that must be allocated to band $\mathcal{B}_0$ in Case II such as to maximize the EE, is higher than the number of bits allocated to band $\mathcal{B}_0$ in Case I.

Finally, in Fig. 3 we present the achievable EE as a function of the number of bands $M$ that the system user can exploit my means of CA. For these simulations, we have considered the system parameters of Case I, i.e. we have selected the fading statistics on bands $\mathcal{B}_m, m = 0, \ldots, M$, to be identical, i.e., it holds that $\bar{g}_0 = \ldots = \bar{g}_M = 1$, and that $B_0 = \ldots = B_M = 1Hz$. Concerning the probability $\pi_1$ that band $m, m = 1, \ldots, M$, is available, two different values for this probability ($\pi_1 = 0.4$ and $\pi_1 = 0.6$) have been considered. Moreover, the results shown in Fig. 3 have been obtained by examining two different constraint values, i.e. $N_b = 10$ and $N_b = 15$, on the total number of feedback bits allocated to bands $\mathcal{B}_0, \ldots \mathcal{B}_M$. Moreover, since the fading statistics on bands $\mathcal{B}_1, \ldots, \mathcal{B}_M$, are identical, we have assumed that the number of feedback bits allocated to $\mathcal{B}_m, m = 1, \ldots, M$, is the same and equal to $N_{b,1}^*$, while the number of feedback bits allocated to carrier $\mathcal{B}_0$ is equal to $N_{b,0}^*$. The values $N_{b,0}^*$ and $N_{b,1}^*$ are set by solving the optimization problem

$$\begin{aligned} &\underset{N_{b,0},N_{b,1}}{\text{maximize:}} \;\; \hat{E}_{eff}\left(N_{b,0}, \underbrace{N_{b,1}, \ldots, N_{b,1}}_{M}\right) \\ &\text{subject to:} \;\; N_{b,0} + MN_{b,1} = N_b, N_{b,0}, N_{b,1} \in \mathbb{N}^+ \end{aligned} \tag{23}$$

where $\hat{E}_{eff}\left(N_{b,0}, N_{b,1}, \ldots, N_{b,M}\right)$ is the achievable EE when $N_{b,m}$, bits are allocated for the quantization of band $m, m = 0, \ldots, M$. Again, the achievable EE also depends on the selected quantization codebook which in our case is designed using a Lloyd-Max quantizer.

As a first remark, one can notice that as the number of feedback bits $N_b$ increases, the achievable EE also increases. This is particularly evident for $M \geq 5$. For lower values of $M$, it appears that the achievable EE for the two different values of $N_b$ is almost identical. This is due to the fact that for small values of $M$, a small number of bits, i.e. $N_b = 10$, is sufficient in order represent the CSIT for carriers $\mathcal{B}_m, m = 0, \ldots, M$. Moreover, it is also easy to see that as $\pi_1$ increases, the achievable EE of the system

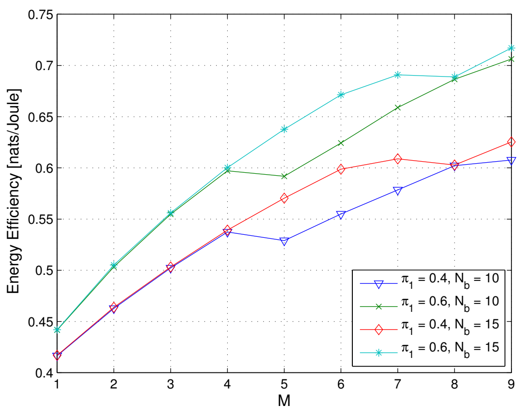


Fig. 3. The optimal EE as a function of the number of PFBs available by means of CA, for different values of $\pi_1$ and $N_b$.

also increases. More importantly though, a very interesting remark that can be noticed is related to the fact that as it can be seen in Fig. 3, in the presence of a constraint on the total number of feedback bits that are allocated to CSI concerning bands $\mathcal{B}_0, \ldots, \mathcal{B}_M$, the achievable EE, is not a monotonically increasing function of the number of PFBs. In more detail, focusing on the case that $N_b = 10$ one can easily see that the achievable EE is slightly higher in the case that $M = 4$ than in the case that $M = 5$. A similar effect also appears in the case that $N_b = 15$ is selected, but this time for higher values of $M$, i.e. $M = 7$. This effect is explained by the fact that as the number of carriers $M$ increases, in the presence of a total constraint on the number of feedback bits, the quality of feedback degrades. Thus, in the presence of a total constraint on the number of feedback bits, increasing the number of PFBs does not necessarily increase the achievable EE. Nevertheless, this effect in monotonicity is eliminated for higher values of $M$. This is due to the fact that for high values for the number of PFBs, the effects of diversity become more and more important, and mitigate the effects of less accurate CSIT. Furthermore, as $M$ increases, it is easy to see that the probability that $k \leq M$ PFBs are indeed available for transmission also increases. This influences the achievable rate and thus the achievable EE.

## VI. Conclusions

The problem of EE-optimal power allocation in CA enabled systems was studied. Both cases of perfect and quantized feedback acquisition at Tx have been considered and related algorithms have been derived. Capitalizing on the derived algorithms, the problems of optimal feedback allocation in CA enabled systems along with the problem of optimally selecting the number of component carriers have been studied and, by means of numerical results, it has been found that the number of feedback bits that should be allocated to each one of the PFBs is a function of fading conditions. Moreover, it has been found that, in the presence of a constraint on the total number of feedback bits allocated to the PFBs, the achievable energy efficiency is not a monotonically increasing function of the number of PFBs. Thus, in the presence of a constraint on the number of feedback bits, caution should be taken when selecting the number of PFBs that should be used.

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## References


[1] A. Goldsmith, *Wireless Communications*. Cambridge University Press, 2005.

[2] A. Ghasemi and E. S. Sousa, “Fundamental limits of spectrum-sharing in fading environments,” *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.

[3] R. Zhang, “Optimal power control over fading cognitive radio channel by exploting primary user CSI,” in *IEEE Global Communications Conference (GLOBECOM)*, 2008, pp. 1–5.

[4] G. A. Ropokis, D. Gesbert, and K. Berberidis, “Rate optimal power policies in underlay cognitive radios with limited feedback,” in *IEEE International Symposium on Personal, Indoor and Mobile Radio Communication (PIMRC)*, Sep. 2013, pp. 740–744.

[5] G. A. Ropokis, C. G. Tsinos, M. C. Filippou, K. Berberidis, D. Gesbert, and T. Ratnarajah, “Joint Power and Sensing Optimization for Hybrid Cognitive Radios with limited CSIT,” in *European Wireless Conference (EW)*, 2015, pp. 1–7.

[6] L. Musavian and T. Le-Ngoc, “Energy-Efficient Power Allocation Over Nakagami-m Fading Channels Under Delay-Outage Constraints,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 8, pp. 4081–4091, Aug. 2014.

[7] “Requirement for further advancements for E-UTRA (LTE-Advanced),” 3GPP TR 36.913 V10.0.0, Tech. Rep., Mar 2011. [Online]. Available: http://www.3gpp.org

[8] F. Liu, K. Zheng, W. Xiang, and H. Zhao, “Design and Performance Analysis of An Energy-Efficient Uplink Carrier Aggregation Scheme,” *IEEE J. Sel. Areas Commun.*, vol. 32, no. 2, pp. 197–207, Feb. 2014.

[9] K. Zheng, F. Liu, W. Xiang, and X. Xin, “Dynamic downlink aggregation carrier scheduling scheme for wireless networks,” *IET Communications*, vol. 8, no. 1, pp. 114–123, Jan. 2013.

[10] L. Xingqin, J. G. Andrews, R. Ratasuk, B. Mondal, and A. Ghosh, “Carrier aggregation in heterogeneous cellular networks,” in *IEEE International Conference on Communications (ICC)* , Jun. 2013, pp. 5199–5203.

[11] F. Kaltenberger, F. Foukalas, O. Holland, S. Pietrzyk, S. Thao, and G. Vivier, “Spectrum overlay through aggregation of heterogeneous dispersed bands ,” in *European Conference on Networks and Communications (EuCNC)* , Jun. 2014, pp. 1–5.

[12] G. Berardinelli, T. B. Sorensen, P. Morgensen, and K. Pajukoski, “Transmission over Multiple Component Carriers in LTE-A Uplink,” *IEEE Com. Magazine*, vol. 18, no. 4, pp. 67–73, Aug. 2011.

[13] D. J. Love, R. W. Heath, V. K. N. Lau, D. Gesbert, B. D. Rao, and M. Andrews, “An overview of limited feedback in wireless communications systems,” *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341–1365, Oct. 2008.

[14] J. Mo and R. W. Heath, “Limited feedback in muliple-antenna systems with one-bit quantization,” in *Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, USA, Nov. 2015.

[15] H. A. Suraweera, P. J. Smith, and M. Shafi, “Capacity Limits and Performance Analysis of Cognitive Radio With Imperfect Channel Knowledge,” *IEEE Trans. Veh. Technol.*, vol. 59, no. 4, pp. 1811–1822, May 2010.

[16] F. Yuan and C. Yang, “Joint power and feedback bit allocation for energy-efficient design in limited-feedback coordinated beamforming systems,” *EURASIP Journal on Wireless Communications and Networking*, vol. 2015, no. 1, May 2015.

[17] B. Makki and T. Eriksson, "Fairness, power allocation, and CSI quantization in block fading multiuser systems," *EURASIP Journal onWireless Communications and Networking*, vol. 2013, no. 1, Oct. 2013.

[18] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical functions with Formulas, Graphs and Mathematical tables*. New York: Dover publications, 1964.

[19] J. Chen, S. Yang, A. Ozgur, and A. Goldsmith, "Achieving full DoF in heterogeneous parallel broadcast channels with outdated CSIT," *Arxiv preprint*. [Online]. Available: http://arxiv.org/pdf/1409.68.08.pdf

[20] S. Schaible, "Minimization of ratios: Technical notes," *J. Optimization Theory Appl.*, vol. 19, no. 2, pp. 347–352, Jun. 1976.

[21] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[22] I. S. Gradshteyn and I. M. Ryzhik, *Tables of integrals, series and products*, 7th ed. Academic Press, 2007.