Optimal Sensing and Power Allocation in Pilot-Aided Shared Access Systems: A BER Minimization Approach

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***Abstract*—We investigate the performance of a hybrid, opportunistic/underlay Licensed Shared Access (LSA) system, in the presence of Spectrum Sensing (SS) and Channel Estimation (CE) uncertainties, and derive a simple closed form expression for the system's Bit Error Rate (BER). Based on the derived expression, we introduce an optimization problem for optimally selecting the time allocated to the SS and CE tasks, as well as the transmit power allocation, such as to minimize the BER of the LSA licensee system, given an average transmit power constraint. It is interestingly observed that the optimal SS time is a decreasing function of the duty cycle of incumbent communication and increases as the system becomes more interference-prone.**

I. Introduction

Spectrum sharing has appeared as a promising technology for tackling spectrum scarcity. Among the several spectrum sharing techniques, Licensed Shared Access (LSA) has recently emerged as a framework respecting QoS requirements for all competing wireless systems [1], [2]. The LSA paradigm introduces two categories of wireless users, namely, the *incumbent* users, that are similar to Primary Users (PUs) in Cognitive Radio (CR) systems, and *licensee* users, which resemble Secondary Users (SUs) in CR systems, with the major difference being in that LSA licensees are granted access by the incumbent, according to a predefined QoS provisioning agreement.

LSA deployments can be realized by either constraining licensee transmissions to geographical areas located sufficiently far from incumbent coverage areas, or by adopting more *flexible* shared access frameworks, where both incumbents and licensees can operate in the same area. In the latter case, Spectrum Sensing (SS) plays a key role, allowing licensees to detect the presence/absence of incumbent activity, in order to appropriately adapt their transmission parameters following communication techniques proposed within the context of hybrid CR networks, as in [3]–[5]. An important characteristic of these techniques is that they focus on the achievable licensee rate as a performance metric for designing/optimizing the system parameters. Moreover, they assume that perfect Channel State Information (CSI) is available to the licensee. Nonetheless, in practice, perfect CSI is not available to the receivers. Hence, the latter must rely on Channel Estimation (CE). Among the several CE algorithms, pilot-aided CE is of particular popularity. In the literature, several works consider legacy, i.e., non spectrum sharing based communication networks, operating in the presence of CSI uncertainties. For example, in [6], [7], closed form Bit Error Rate (BER) and Symbol Error Rate (SER) expressions are derived for legacy systems in the presence of channel estimation errors. Focusing on CR/LSA networks, the effects of imperfect CSI must be investigated jointly with the effects of imperfect SS. Such an approach is presented in [8], where channel estimators for CR/LSA systems subject to SS errors are derived. More importantly, in [9] expressions are derived for the achievable rate of Single-Input Single-Output (SISO) CR systems in the presence of SS and CE uncertainties for Gaussian signaling. However, for the more realistic case of linear modulation, no closed form expressions are available.

Motivated by the above, in this work we focus on analyzing the performance of flexible LSA systems, in terms of the achievable BER, in the presence of both CSI and SS uncertainties. The contribution of this work is twofold: First, assuming Binary Phase Shift Keying (BPSK) data transmission, we derive a closed form expression for the achievable BER of hybrid shared access systems operating in the presence of SS and CE errors over Rayleigh fading channels. Then, capitalizing on the derived BER expression, we explore the problem of optimizing the power allocation at the LSA licensee, along with the problem of optimally selecting SS and CE time durations, such as to minimize the licensee system BER. We observe that the BER-optimal SS time decreases for increasing probability of incumbent activity, while it increases as the system becomes more interference-limited.

Notation: The $M \times M$ identity matrix is denoted as $\mathbf{I}_M$ and the all-zero $M \times L$ matrix as $\mathbf{0}_{M\times L}$. Notation $\mathbf{a}\left(m:n\right), n \geq m$, denotes the vector that is formed by taking elements $m, m+1, \ldots, n$ of vector $\mathbf{a}$. Operator $\otimes$ stands for the Kronecker product. Notation $\mathbf{x} \sim \mathcal{CN}\left(\mathbf{0}_{M\times 1}, \mathbf{R}_{\mathbf{x}}\right)$ denotes

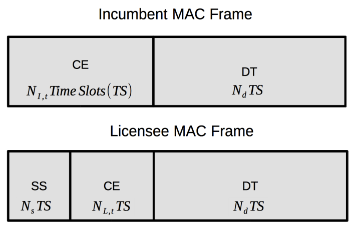

Fig. 1. MAC frame structure of the examined LSA system.

that $\mathbf{x}$ follows a zero mean Circularly Symmetric Complex Gaussian (CSCG) distribution with covariance matrix $\mathbf{R_x}$. The expectation of a Random Variable (RV) is denoted as $\mathbb{E}\{\cdot\}$. The exponential integral, [10, eq. (5.1.1)], is denoted as $E_1(\cdot)$. Finally, $\Gamma(\cdot,\cdot)$ and $\Gamma(\cdot)$ denote the upper incomplete Gamma and the Gamma functions respectively [10, eqs. (6.1.1) and (6.5.2-3)], and $Q_K(\cdot,\cdot)$ stands for the Marcum-Q function [11].

II. System model

We study a Single-Input Multiple-Output (SIMO) licensee system with M receive antennas, that operates in the presence of an incumbent system. Both networks transmit BPSK modulated data, while we further assume that incumbent communication takes place following a specific duty cycle which is known to the licensee. That is, if $\mathcal{H}_0$ denotes the event of incumbent absence, and $\mathcal{H}_1$ its complementary, the Licensee Transmitter (LTx) has knowledge of probabilities $\mathcal{P}_0 = \Pr(\mathcal{H}_0)$ and $\mathcal{P}_1 = 1 - \mathcal{P}_0$. In addition, we assume that both the incumbent and licensee systems have a similar Medium Access Control (MAC) frame structure. This means that both MAC frames comprise of N Time Slots (TSs), each of duration of T seconds, where T is also the duration of one BPSK symbol. Moreover, without loss of generality and in order to simplify our analysis, the incumbent and licensee MAC frames are deemed to be perfectly synchronized.

The licensee system employs SS, in order to detect the presence/absence of an incumbent user. In particular, we investigate a system case where the MAC frame of the licensee system is split into three subframes, as shown in Fig. 1. The first subframe consists of N_s TSs, during which SS takes place. This subframe is followed by a second one, of size of $N_{L,t}$ TSs, during which training symbols are transmitted so that the Licensee Receiver (LRx) can estimate the channel between LTx and LRx. We denote by $\mathbf{s}_{L,t} = \left[s_{L,t,1}, \ldots, s_{L,t,N_{L,t}}\right]^T$ this BPSK training sequence. The rest of the MAC frame, that is of duration of N_d time slots, is used for Data Transmission (DT) from LTx to LRx. The packet of data transmitted during this subframe is denoted by $\mathbf{s}_{L,d} = \left[s_{L,d,1}, \ldots, s_{L,d,N_d}\right]^T$. The incumbent communication MAC frame is split into two subframes. In the first subframe, of duration of $N_{I,t} = N_s + N_{L,t}$ time slots, the incumbent system performs CE, in order to estimate the Incumbent Tx (ITx) to Incumbent Receiver (IRx) channel, by employing the BPSK training sequence $\mathbf{s}_{I,t} = \left[s_{I,t,1}, \ldots, s_{I,t,N_{I,t}}\right]^T$. Then, a data packet $\mathbf{s}_{I,d} = \left[s_{I,d,1}, \ldots, s_{I,d,N_d}\right]^T$ of size N_d is transmitted from ITx to IRx during the second MAC subframe. Due to synchronization between the incumbent and licensee systems, we assume that the SS and CE subframes of the licensee MAC frame are aligned in time with the CE subframe of the incumbent MAC frame, while the DT subframe of the licensee MAC frame is perfectly synchronized with the DT subframe of the incumbent MAC frame as shown in Fig. 1. Moreover, we consider that training sequences $\mathbf{s}_{I,t}$ and $\mathbf{s}_{L,t}$, used by the incumbent and licensee systems, respectively, are fixed and known to both IRx and LRx. Focusing on the licensee MAC frame, the following subsections further describe the operation mode during its three subframes.

A. Licensee Spectrum Sensing

We utilize Energy Detection (ED) SS, due to its implementation simplicity and analytical tractability. As in [5], we assume that SS takes place at LTx, where k measurements are taken in each time slot (TS) for ED. Hence, the total number of samples, K, in the ED subframe is $K = kN_s$. The ED decision variable is then expressed as

$$\mathcal{E} = \frac{2\sum_{n=1}^{K}\left|U(\mathcal{H}_1)\,h_t\sqrt{P_{I,t}}s_{I,t,\lceil\frac{n}{k}\rceil} + w_n\right|^2}{N_0}, \quad (1)$$

where $w_n \sim \mathcal{CN}(0, N_0)$ stands for the Additive White Gaussian Noise (AWGN) at LTx, and $h_t \sim \mathcal{CN}\left(0, \sigma_t^2\right)$ stands for the ITx-LTx Rayleigh fading channel. Moreover $P_{I,t}$ stands for the power of the symbols transmitted by ITx during its training subframe. Finally $U(\mathcal{H}_1)$ is an indicator function, equal to one when event $\mathcal{H}_1$ occurs and zero otherwise.

By comparing $\mathcal{E}$ with a predefined threshold ε, ED decides upon the absence/presence of an incumbent transmission. Let $\hat{\mathcal{H}}_l, l \in \{0,1\}$ denote the event that ED has decided upon event $\mathcal{H}_l$. The most important performance figures of merit of the SS algorithm are the detection probability $\mathcal{P}_d(\varepsilon, K) = \Pr(\mathcal{E} \geq \varepsilon | \mathcal{H}_1, \varepsilon, K)$ and the False Alarm (FA) probability $\mathcal{P}_f(\varepsilon, K) = \Pr(\mathcal{E} \geq \varepsilon | \mathcal{H}_0, \varepsilon, K)$. Following a procedure similar to the one presented in [11], $\mathcal{P}_d(\varepsilon, K)$ is expressed as

$$\mathcal{P}_d(\varepsilon, K) = \int_0^\infty \mathcal{Q}_K\left(\sqrt{2K\gamma}, \sqrt{\varepsilon}\right)\exp\left(-\frac{\gamma}{\bar{\gamma}_t}\right)d\gamma, \quad (2)$$

where γ stands for the instantaneous Signal to Noise Ratio (SNR), $\bar{\gamma}_t = \mathbb{E}\{\gamma\} = \sigma_t^2 P_{I,t}/N_0$, and $\mathcal{P}_f(\varepsilon, K)$ as $\mathcal{P}_f(\varepsilon, K) = \frac{\Gamma(K, \varepsilon/2)}{\Gamma(K)}$. This result can be derived by noticing that given h_t and the presence/absence of incumbent transmission, the ED decision variable $\mathcal{E}$ is a noncentral/central chi squared RV with $2K$ degrees of freedom. Furthermore, integral (2) can be written in closed form by employing [11, eq. (9)].

B. Licensee Channel Estimation

During the CE subframe, the training sequence $\mathbf{s}_{L,t}$ is transmitted. Let $P_{L,t}$ be the transmit power per symbol. Then, the signal received by LRx in the i-th TS of the licensee MAC training subframe is written as

$$\mathbf{y}_{t,i} = \mathbf{h}_{LL}\sqrt{P_{L,t}}s_{L,t,i} + U\left(\mathcal{H}_1\right)\mathbf{h}_{IL}\sqrt{P_{I,t}}s_{I,t,i+N_s} + \mathbf{n}_{t,i}, \tag{3}$$

where $\mathbf{h}_{LL}$ and $\mathbf{h}_{IL}$ denote the LTx-LRx and ITx-LRx channels, respectively. In our analysis we adopt an independent and identically distributed (i.i.d.) Rayleigh fading channel model, i.e., $\mathbf{h}_{LL} \sim \mathcal{CN}\left(\mathbf{0}_{M\times 1}, \sigma_{LL}^2\mathbf{I}_M\right)$ and $\mathbf{h}_{IL} \sim \mathcal{CN}\left(\mathbf{0}_{M\times 1}, \sigma_{IL}^2\mathbf{I}_M\right)$. Moreover, we assume that $\mathbf{h}_{LL}$ and $\mathbf{h}_{IL}$ are mutually independent and remain constant for the whole duration of a MAC frame. Finally, $\mathbf{n}_{t,i} \sim \mathcal{CN}\left(\mathbf{0}_{M\times 1}, N_0\mathbf{I}_M\right)$ stands for AWGN at LRx during the training subframe.

In our analysis, LRx uses a simple Least Squares (LS) channel estimator. Hence, by defining $\mathbf{y}_t = \left[\mathbf{y}_{t,1}^T, \ldots, \mathbf{y}_{t,N_{L,t}}^T\right]^T$ and $\mathbf{n}_t = \left[\mathbf{n}_{t,1}^T, \ldots, \mathbf{n}_{t,N_{L,t}}^T\right]^T$, it then holds that

$$\mathbf{y}_t = \mathbf{S}_{L,t}\mathbf{h}_{LL}\sqrt{P_{L,t}} + U\left(\mathcal{H}_1\right)\sqrt{P_{I,t}}\mathbf{S}_{I,t}\mathbf{h}_{IL} + \mathbf{n}_t, \tag{4}$$

where $\mathbf{S}_{L,t} = \mathbf{s}_{L,t}\otimes\mathbf{I}_M$, and $\mathbf{S}_{I,t} = \mathbf{s}_{I,t}\left(N_s+1 : N_{I,t}\right)\otimes\mathbf{I}_M$. The LS channel estimate is then written as

$$\hat{\mathbf{h}}_{LL} = \frac{\mathbf{S}_{L,t}^H\mathbf{y}_t}{\sqrt{P_{L,t}}N_{L,t}} = \mathbf{h}_{LL} + U\left(\mathcal{H}_1\right)\zeta\mathbf{h}_{IL} + \frac{\mathbf{S}_{L,t}^H\mathbf{n}_t}{\sqrt{P_{L,t}}N_{L,t}}, \tag{5}$$

where we have used the fact that $\mathbf{S}_{L,t}^H\mathbf{S}_{L,t} = N_{L,t}\mathbf{I}_M$. In (5), ζ is defined as $\zeta = \frac{1}{N_{L,t}}\sqrt{\frac{P_{I,t}}{P_{L,t}}}\sum_{j=1}^{N_{L,t}} s_{L,t,j}^H s_{I,t,j+N_s}$. Given synchronization as well as the specific structure for the incumbent and licensee MAC frames, we select training sequences $\mathbf{s}_{L,t}$ and $\mathbf{s}_{I,t}$ such that $\zeta = 0$. Note that in doing so, the effects of pilot contamination during estimation of $\mathbf{h}_{LL}$ at LRx as well as during estimation of the ITx-IRx channel at IRx are avoided. On the other hand, it should be remarked that ensuring that $\zeta = 0$, restricts the potential values of $N_{L,t}$ to the set of even integers. Finally, given the fact that no pilot contamination is present, the LTx transmission during the CE subframe does not interfere with the CE performed by IRx, and thus we assume that LTx employs all its available power in the CE phase, i.e., it sets $P_{L,t} = P_{max}$.

C. Licensee Data Transmission

In the DT subframe, the power level used by LTx depends upon the SS decision. That is, LTx sets the transmit power to $P_{L,d} = P_l$, provided that, as a result of SS, event $\hat{\mathcal{H}}_l, l \in \{0,1\}$ occurs. For the case that no incumbent activity is detected, the power level P_0 is selected to be equal to the maximum allowable transmit power P_{max}. On the other hand, in case that incumbent activity is detected, following a typical underlay approach, the power level is set as

$$P_1 = \min\left\{\mathcal{I}/\left|\hat{h}_{LI}\right|^2, P_{max}\right\}, \tag{6}$$

where $\mathcal{I}$ is a threshold set on the instantaneous interference caused by the licensee system to the incumbent system, and $\hat{h}_{LI}$ is an estimate for the interference channel between LTx and IRx[1]. Channel estimate $\hat{h}_{LI}$, can be considered as a noisy version of the actual LTx-IRx channel h_{LI}, i.e., it holds that

$$\hat{h}_{LI} = h_{LI} + e_{LI}, h_{LI} \sim \mathcal{CN}\left(0, \sigma_{LI}^2\right), e_{LI} \sim \mathcal{CN}\left(0, \sigma_e^2\right). \tag{7}$$

Thus, $g_{LI} = \left|\hat{h}_{LI}\right|^2$ is an exponential RV with mean $\hat{\sigma}_{LI}^2 = \sigma_{LI}^2 + \sigma_e^2$. Concerning availability of $\hat{h}_{LI}$ at LTx, this can be achieved either by periodical transmission of this information from IRx to LTx. Alternatively, such information can be obtained if LTx overhears the transmission of pilot signals from IRx to ITx and applies channel estimation.

The signal received at LRx in the j-th TS of the DT subframe, $j = 1, \ldots, N_d$, provided that SS has decided in favor of event $\hat{\mathcal{H}}_l$, is expressed as

$$\mathbf{y}_{d,j} = \mathbf{h}_{LL}\sqrt{P_l}s_{L,d,j} + U\left(\mathcal{H}_1\right)\mathbf{h}_{IL}\sqrt{P_{I,d}}s_{I,d,j} + \mathbf{n}_{L,j}, \tag{8}$$

where $P_{I,d}$ stands for the per symbol transmit power of the incumbent system during the DT subframe and $\mathbf{n}_{I,j} \sim \mathcal{CN}\left(\mathbf{0}_{M\times 1}, N_0\mathbf{I}_M\right)$ stands for the AWGN. We assume that the receiver employs Maximum Ratio Combining (MRC). Thus, the decision variable used in order to detect the BPSK symbol $s_{L,d,j}$ is expressed as

$$d_j = \Re\left\{\hat{\mathbf{h}}_{LL}^H\mathbf{y}_{d,j}\right\} = \mathbf{x}_j^H \underbrace{\frac{1}{2}\begin{bmatrix}\mathbf{0}_{M\times M} & \mathbf{I}_M \\ \mathbf{I}_M & \mathbf{0}_{M\times M}\end{bmatrix}}_{\mathbf{A}}\mathbf{x}_j, \tag{9}$$

where $\mathbf{x}_j = \left[\hat{\mathbf{h}}_{LL}^T, \mathbf{y}_{d,j}^T\right]^T$. Based on d_j, demodulation is performed and the value $\hat{s}_{L,d,j}$ is selected for the j-th data symbol, according to the rule $\hat{s}_{L,d,j} = \text{sign}\left(d_j\right)$. We are interested in obtaining an expression for the BER during the detection of $s_{L,d,j}$ for the described system model. The procedure for calculating the BER is presented in Section III.

III. Licensee BER derivation

Without loss of generality, we assume that the transmitted symbol is $s_{L,d,j} = 1$. Thus, taking also into account the form of the applied power policy scheme on the occurrence of events $\hat{\mathcal{H}}_l, l \in \{0,1\}$, one can express the BER of the licensee system as

$$P_b = \sum_{l=0}^{1}\sum_{r=0}^{1}\Pr\left(\hat{\mathcal{H}}_l, \mathcal{H}_r\right)\mathbb{E}_{g_{LI}}\left\{\Pr\left(d_j \leq 0|\mathcal{H}_r, \hat{\mathcal{H}}_l, g_{LI}\right)\right\}. \tag{10}$$

The calculation of the BER then reduces to the calculation of the Cumulative Distribution Function (CDF) of RV d_j at zero under the conditional knowledge of events $\mathcal{H}_r, \hat{\mathcal{H}}_l$ $r, l \in \{0,1\}$ and $g_{LI} = g$. For the calculation of this probability, let us start by noting that d_j in (9) is a Hermitian Quadratic Form (QF) in RVs. Moreover, it is easy to show that, given the occurrence of events $\mathcal{H}_r, \hat{\mathcal{H}}_l, r, l \in \{0,1\}$ and $g_{LI} = g$, $\mathbf{x}_j$ is a complex Gaussian random vector with a covariance matrix $\mathbf{R}_{\mathbf{x}_j|\mathcal{H}_r,\hat{\mathcal{H}}_l} = \mathbf{R}\left(\alpha, \beta_l, \delta_{r,l}\right)$, where

$$\mathbf{R}\left(\alpha, \beta_l, \delta_{r,l}\right) = \begin{bmatrix}\alpha\mathbf{I}_M & \beta_l\mathbf{I}_M \\ \beta_l\mathbf{I}_M & \delta_{r,l}\mathbf{I}_M\end{bmatrix}, \tag{11}$$

[1]For simplicity a SISO incumbent system is considered.

with $\alpha = \sigma_{LL}^2 + \frac{N_0}{N_{L,t} P_{L,t}}$, $\beta_l = \sqrt{P_l}\sigma_{LL}^2$, $\delta_{0,l} = P_l \sigma_{LL}^2 + N_0$, $\delta_{1,l} = P_l \sigma_{LL}^2 + P_{I,d}\sigma_{IL}^2 + N_0$, $l \in \{0,1\}$. Thus, given that events $\mathcal{H}_r, \hat{\mathcal{H}}_l, r,l \in \{0,1\}$, have occurred, d_j is expressed as a Hermitian QF in complex normal RVs, and its Moment Generating Function (MGF) is given as [12]

$$\mathcal{M}_{d_j|\mathcal{H}_r,\hat{\mathcal{H}}_l}(t) = \prod_{i=1}^{2M} \frac{1}{\left(1 - \lambda_{i|\mathcal{H}_r,\hat{\mathcal{H}}_l} t\right)}, \tag{12}$$

where $\lambda_{1|\mathcal{H}_r,\hat{\mathcal{H}}_l}, \dots, \lambda_{2M|\mathcal{H}_r,\hat{\mathcal{H}}_l}$ are the eigenvalues of matrix $\mathbf{R}_{\mathbf{x}_j|\mathcal{H}_r,\hat{\mathcal{H}}_l}\mathbf{A}$. From the form of matrices $\mathbf{A}$ and $\mathbf{R}_{\mathbf{x}_j|\mathcal{H}_r,\hat{\mathcal{H}}_l}$, and employing [13, pp. 66. Property 17], it is easy to show that matrix $\mathbf{R}_{\mathbf{x}_j|\mathcal{H}_r,\hat{\mathcal{H}}_l}\mathbf{A}$ has only two eigenvalues, given as $\rho_{j,i|\mathcal{H}_r,\hat{\mathcal{H}}_l} = \frac{\beta_l + (-1)^{i-1}\sqrt{\alpha\delta_{r,l}}}{2}, r,l \in \{0,1\}$ each one with multiplicity M. Employing this result in (12), the MGF of d_j is rewritten as

$$\mathcal{M}_{d_j|\mathcal{H}_r,\hat{\mathcal{H}}_l}(t) = \sum_{i=1}^{2}\sum_{m=1}^{M} \frac{A_{j,i,M-m+1|\mathcal{H}_r,\hat{\mathcal{H}}_l}}{\left(1 - \rho_{j,i|\mathcal{H}_r,\hat{\mathcal{H}}_l} t\right)^m}. \tag{13}$$

Coefficients $A_{j,i,m|\mathcal{H}_r,\hat{\mathcal{H}}_l}$'s in (13) , are the Partial Fraction Expansion (PFE) coefficients of $\mathcal{M}_{d_j|\mathcal{H}_r,\hat{\mathcal{H}}_l}(t)$. Based on (13), and the expression given in [12] for the PFE coefficients of (13), it is easy to show that

$$\Pr\left(d_j \leq 0 | \mathcal{H}_r, \hat{\mathcal{H}}_l, g_{LI} = g\right) = \sum_{m=1}^{M} A_{j,2,m|\mathcal{H}_r,\hat{\mathcal{H}}_l} = \left(\frac{1-\mu_{r,l}}{2}\right)^M \sum_{m=1}^{M} \binom{M-1+(m-1)}{m-1} \left(\frac{1+\mu_{r,l}}{2}\right)^{m-1}, \tag{14}$$

where $\mu_{r,l} = \beta_l / \sqrt{\alpha\delta_{r,l}}$. Clearly, in case of event $\hat{\mathcal{H}}_1$, since power level P_1 depends on the value of g_{LI}, $\mu_{r,1}$ is a function of g_{LI}. On the other hand, in case of event $\hat{\mathcal{H}}_0$, $\mu_{r,0}$ is independent of g and thus $\Pr\left(d_j \leq 0 | \mathcal{H}_r, \hat{\mathcal{H}}_0, g_{LI} = g\right) = \Pr\left(d_j \leq 0 | \mathcal{H}_r, \hat{\mathcal{H}}_0\right)$. Using (14), the BER in (10) can be computed by taking into account the exponential form of the Probability Density Function (PDF) of g_{LI}, and applying Gauss-Laguerre quadrature rules [10, eq. (25.4.45), pp. 890] to calculate the expectations in (10).

IV. BER OPTIMAL SS, CE AND DT

Having derived a closed form expression for the BER in (10), in this section we exploit this result to investigate the optimal design of the SS, CE and DT subframes. More specifically, we are interested in the SS time selection and power allocation such as to minimize the licensee system BER, subject to a power budget constraint, P. This optimization problem can be expressed as

$$\begin{aligned} &\underset{N_s, N_{L,t}, P_{max}}{\text{minimize}} \quad P_b \\ &\text{subject to: } \mathcal{P}_d = \tilde{\mathcal{P}}_d, \\ &P_{max} \geq 0,\ N_s + N_{L,t} = N_{I,t},\ N_s \geq 1,\ N_s, \frac{N_{L,t}}{2} \in \mathbb{N}, \\ &\frac{N_{L,t} P_{max}}{N} + \frac{N_d \left(\Pr\left(\hat{\mathcal{H}}_0\right) P_{max} + \Pr\left(\hat{\mathcal{H}}_1\right) \mathbb{E}\{P_1\}\right)}{N} = P. \end{aligned} \tag{15}$$

The introduction of the power budget constraint in (15) allows to fairly compare systems that are characterized by different values for the SS time by treating P_{max} as an additional parameter, selected such as to satisfy the power budget constraint. Notice that the optimization problem in (15) takes also into account an equality constraint $\tilde{\mathcal{P}}_d$ on the achievable detection probability, selected high enough so that incumbent transmission is reliably detected and protected. By employing (2), one can numerically evaluate the decision threshold $\varepsilon(N_s)$ for which, for any value of N_s, constraint $\mathcal{P}_d = \tilde{\mathcal{P}}_d$ is satisfied.

To solve the problem in (15), evaluation of $\mathbb{E}\{P_1\}$ is required. Exploiting that RV g_{LI} is an exponential RV and using [10, eq. (5.1.1), pp. 228], $\mathbb{E}\{P_1\}$ can be expressed as

$$\mathbb{E}\{P_1\} = \Pr\left(g_{LI} \leq \frac{I}{P_{max}}\right) P_{max} + \frac{I}{\hat{\sigma}_{LI}^2} E_1\left(\frac{I}{\hat{\sigma}_{LI}^2 P_{max}}\right). \tag{16}$$

Hence, the optimization problem can be easily solved by applying an exhaustive search with respect to the optimal values of N_s and $N_{L,t}$ that satisfy the constraints of (15). In the following section we present numerical results concerning the achievable BER of licensee systems, obtained by solving the optimization problem defined in (15).

V. NUMERICAL RESULTS AND CONCLUSIONS

In this section, we investigate the BER performance of a licensee system operating in the presence of an incumbent system. The key system parameters for the simulations are shown in Table I. For the interference created by ITx to LRx, two different scenarios are investigated. In *Scenario I*, we select σ_{IL}^2 such that the average interference caused by ITx to LRx is equal to 3 dB, while in *Scenario II*, we set σ_{IL}^2 such that the average ITx-LRx interference is equal to 5 dB. Moreover, for both scenarios, for simplicity, we set $\sigma_{LI}^2 = \sigma_{IL}^2$ and $\sigma_e^2 = 0.1\sigma_{LI}^2$. Finally, the average power constraint is set to $P = 10$ dB. In Fig. 2 the achievable licensee system BER calculated using (10) and (14) is plotted as a function of the average licensee system SNR, defined as $\bar{\gamma}_{LL} = \sigma_{LL}^2 P / N_0$, along with Monte Carlo simulation results. The values of the parameters $N_s, N_{L,t}, P_{max}$ for the plotted results, were determined by solving optimization problem (15). It can be seen that the theoretical and simulation results are in full agreement. This confirms the validity of our theoretical analysis.

We now focus on the influence of channel parameters and incumbent activity on the optimal SS time. First, we focus on

TABLE I
SIMULATION PARAMETERS

$\mathcal{P}_1$	$N_0[dB]$	$\bar{\gamma}_t[dB]$	$P_{I,d}[dB]$	N	$N_{I,t}$	k	$\tilde{\mathcal{P}}_d$
0.4	0	0	10	100	10	20	0.9

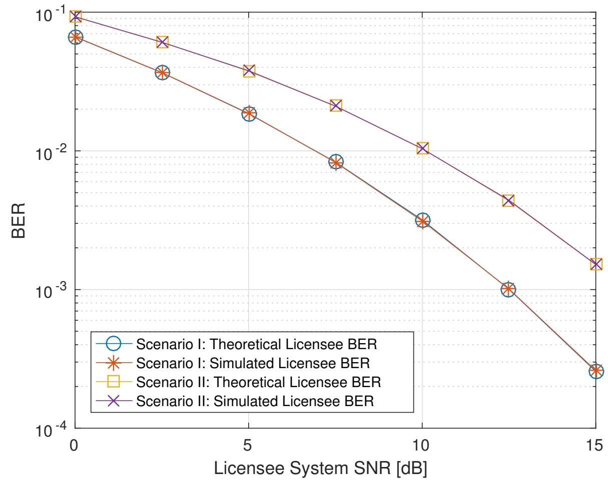

Fig. 2. BER of the licensee system as a function of the average SNR.

the relation between the optimal SS time and the incumbent activity. In order to study this relation, we adopt the system and channel parameters of Scenario II presented earlier, and evaluate the optimal SS time for different values of probability $\mathcal{P}_1$, when $\bar{\gamma}_{LL} = 10$ dB. Table II presents the optimal SS time obtained by solving optimization problem (15) as a function of $\mathcal{P}_1$. It becomes apparent that the optimal SS time is a decreasing function of the probability of incumbent activity. This can be explained by noticing that as $\mathcal{P}_1$ increases, the BER value is dominated by the terms corresponding to event $\mathcal{H}_1$. As a result, BER minimization is achieved by suppressing the terms in (10), that correspond to the BER, given the occurrence of event $\mathcal{H}_1$. This can be done by increasing CE quality and DT quality. While an increase in CE time $N_{L,t}$ could improve CE quality, one should also take into account the fact that by increasing $N_{L,t}$, the time fraction of the MAC frame during which transmission takes place increases. As a result, given the average power constraint, increasing $N_{L,t}$ could result in a decrease in the maximum allowable power level P_{max}. This effect, however, is prevented by the fact that the decrease of sensing time N_s leads to less reliable sensing and increased FA probability which in turn leads to reducing the power proportion allocated to the joint event $\hat{\mathcal{H}}_0, \mathcal{H}_0$. As a result the decrease of P_{max} is avoided. This is also illustrated in Table II, where FA probability and power allocation P_{max} values corresponding to the optimal power and SS time allocation are shown, for different values of $\mathcal{P}_1$.

In Table III, the optimal SS time is depicted as a function of σ^2_{IL} (which coincides with σ^2_{LI}), for an incumbent activity profile $\mathcal{P}_1 = 0.4$ and $\bar{\gamma}_{LL} = 10$ dB. The remaining system parameters are the ones described in Scenario II. One can

TABLE II
OPTIMAL SS TIME, CORRESPONDING FA PROBABILITY AND OPTIMAL POWER ALLOCATION FOR DIFFERENT VALUES OF $\mathcal{P}_1$.

$\mathcal{P}_1$	N_s	$\mathcal{P}_f$	P_{max}
0.2	6	0.15	12.61
0.4	4	0.22	14.10
0.6	4	0.22	16.09
0.8	2	0.34	18.98

TABLE III
OPTIMAL SS TIME, CORRESPONDING FA PROBABILITY AND OPTIMAL POWER ALLOCATION FOR DIFFERENT VALUES OF σ^2_{IL}.

σ^2_{IL} [dB]	N_s	$\mathcal{P}_f$	P_{max}
-10	2	0.75	12.18
-5	4	0.68	19.53
0	6	0.63	28.12
5	8	0.60	33.20

observe that the optimal SS time increases as the strength of interference becomes higher, or equivalently, the CE time reduces when the system becomes more interference-limited. This can be explained by noticing that for a fixed incumbent activity and a fixed detection probability, as in our case, as σ^2_{IL} and σ^2_{LI} increase, the achievable BER is strongly influenced by the fact that in case of FA events the allocated power is on average small, due to the applied power policy given in (6). Therefore, it is crucial to increase the SS time, such as to reduce the FA probability. While this choice could lead to degraded CE quality due to reduced CE time, it also reduces the time proportion during which transmission takes place, allowing for increasing P_{max} while satisfying the average power constraint. This is illustrated in Table III where FA probability and power values P_{max} of the optimal power and SS time allocation are shown, for different values of σ^2_{IL}. Summarizing the presented results, by means of simulations, we have confirmed the validity of our BER expression. Moreover, it has been shown, that the BER-optimal SS time is an increasing function of ITx-LRx interference as well as a decreasing function of the probability of incumbent activity.

VI. ACKNOWLEDGMENTS

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