Cooperative Beamforming Exploiting Energy Recycling

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***Abstract*—We propose and investigate a communications scheme for exploiting multiple Amplify and Forward (AF) relays using the principle of energy recycling. For this scheme, we consider concurrent use of the multiple relays, and study the problem of rate-optimal beamformer design, presenting a new algorithmic solution. We formulate this problem as a Quadratically Constrained Quadratic Ratio (QCQR) programming problem that can be solved exploiting convex optimization techniques. By means of simulations we assess the performance of the proposed solution. Our analysis shows that significant performance improvements can be achieved as compared to single relay solutions.**

***Index Terms*—Energy harvesting, energy recycling, cooperative communications, optimal beamformer design.**

I. Introduction

The problem of optimal beamforming for multi-antenna, wireless powered communications has attracted significant interest and several solutions have been suggested targeting at different optimization criteria and different system designs (see for example [1] and references therein). Extending the works described in [1], recently, [2]–[4], have considered the beamforming problem focusing on wireless powered Full Duplex (FD) relays, with emphasis on Amplify and Forward (AF) based systems. The motivation for these works is that FD techniques allow for the better exploitation of energy resources at the wireless powered relays. This is due to the fact that for the schemes investigated in [2]–[6] the FD relays employ some of the antennas in order to transmit information, and the remaining antennas in order to harvest energy from RF signals reaching these antennas, including signals concurrently transmitted by the relay itself. As a result, those schemes manage to combine energy harvesting and energy recycling. Nevertheless, while all the solutions studied in [2]–[6] provide system designs that exploit the benefits of energy harvesting/ energy recycling, they are limited to single relay systems.

Motivated by this, in this work we seek ways to further improve the performance of energy recycling-based cooperative communications systems, by introducing a scheme that exploits multiple relays. While the use of multi-relay schemes in full-duplex energy-harvesting has been investigated, existing solutions consider two-way relays, i.e., relays where the full-duplex characteristics are employed so that the relays can forward data concurrently on both communications sides that act as sources and destinations simultaneously [7]. In contrast, in this work we consider the case of one-way, multi-antenna, wireless powered relays, exploiting full duplex characteristics in order to achieve concurrent transmission and energy harvesting. For this multi-relay system design, we investigate the problem of rate-optimal beamformer design and propose a new cooperative beamforming solution that is based on the use of Quadratically Constrained Quadratic Ratio (QCQRs) programming techniques. To the best of our knowledge the proposed beamformer is the first to allow for the concurrent utilization of multiple relays exploiting full duplex relays in a manner that allows for concurrent data transmission and energy harvesting in multi-relay scenarios. As a result, our proposed system design manages to convert self-interference from an undesired effect to a factor that can be further exploited in order to increase the potential for energy harvesting. By means of simulations we evaluate the performance of our proposed scheme, and we find that significant performance benefits, as compared to single-relay schemes, can be achieved.

The paper is structured as follows. In Section II, we present our multi-relay cooperative beamforming system model, while in Section III we develop a method for designing the beamformer for this system. Section IV presents the performance comparison for the schemes introduced in this paper with existing single relay solutions. Finally, in Section V, we summarize our conclusions.

Notation: We use lower-case bold letters to denote vectors, and upper case bold letters to denote matrices. $\mathbf{A}^T$ stands for the transpose of matrix $\mathbf{A}$ and $\mathbf{A}^H$ for the hermitian transpose of matrix $\mathbf{A}$. The $M \times M$ identity matrix is denoted as $\mathbf{I}_M$ and the all-zero $M \times 1$ vector as $\mathbf{0}_M$. We use $\|\mathbf{x}\|$ to denote the norm of vector $\mathbf{x}$. We use notation $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}_M, \mathbf{R})$ to indicate that random vector $\mathbf{x}$ follows a complex Gaussian distribution with mean value equal to zero and covariance matrix $\mathbf{R}$. Operator $(\cdot)^*$ is used to denote complex conjugation.

II. System model

We consider a cooperative system comprising a source S with a single transmit antenna, that communicates with a destination D equipped with one receive antenna. Communication is achieved exploiting a set of L full-duplex relays, $\{R_1, \ldots, R_L\}$ each one equipped with $M + 1$ antennas, and powered by means of energy harvesting. In more detail, in our system model relay $R_l, l = 1, \ldots, L$, harvests energy transmitted by S, energy transmitted by its own transmit antennas, and energy from the remaining relays. We assume that the $S \rightarrow D$ link is absent, necessitating the use of the $S \rightarrow R_l \rightarrow D$ links in order to assist communication.

Focusing on relay $R_l, l = 1, \ldots, L$, one of its antennas is devoted to receiving data and energy from the source S, other relays, and from itself. The remaining antennas of R_l are devoted to relaying information to D, employing an Amplify and Forward (AF) scheme. Each time slot t, of duration T, is split in two phases, each of duration equal to $T/2$. The system operation during each of these phases is described in what follows, and summarized in Table I.

TABLE I
THE SYSTEM OPERATION IN THE TWO PHASES OF THE COOPERATIVE BEAMFORMING SCHEME

	Phase I	Phase II
Source	S transmits data	S transmits energy
Relays	Relays receive the transmitted data	Relays forward data received during phase I using their transmit antennas and harvest energy through their receive antenna

A. Phase I: Data transmission from S

In the first phase of time slot t, the source S transmits data, which is received by the relays. The signal received at relay R_l in this phase is expressed as

$$y_{r,l,1}[t] = h_{sr,l}[t] \sqrt{P_s} x_{s,1}[t] + n_{r,l,1}[t], \tag{4}$$

where P_s is the transmit power of S, $h_{sr,l}[t]$ characterizes the $S \rightarrow R_l$ channel, $x_{s,1}[t]$ is the signal transmitted by S during this phase, and $n_{r,l,1}[t] \sim \mathcal{CN}(0, N_0)$, is the Additive White Gaussian Noise (AWGN) at R_l, with noise density N_0.

B. Phase II: Data transmission/energy harvesting from/at R

In the second phase of time slot t, the signal received by $R_l, l = 1, \ldots, L$, during the first phase is normalized and amplified as in [8] and forwarded to D using a power level $P_{r,l}[t]$. Concurrently, another signal $x_{s,2}[t]$ is transmitted by S, to be exploited by $R_l, l = 1, \ldots, L$, for energy harvesting purposes. The signal reaching D is then written as in (1), where $\mathbf{h}_{rd,l}[t]$ denotes the $M \times 1$, $R_l \rightarrow D$ channel while $n_{d,2}[t] \sim \mathcal{CN}(0, N_0)$ is the AWGN at D. Note that quantities $\alpha_l[t]$ and $\beta_l[t]$ account for the power normalization that takes place at the relay. Finally $\mathbf{w}_l[t]$, where $\|\mathbf{w}_l[t]\|^2 = P_{r,l}[t]$, is the $M \times 1$ beamformer applied at the M antennas used for transmission at R_l, with $P_{r,l}[t]$ being the power level employed by R_l.

Focusing on the energy harvesting operation performed at R_l, we assume that R_l harvests the energy reaching its receive antenna during phase II. This consists of energy carried by signal $x_{s,2}[t]$, transmitted by S, as well as energy transmitted by the transmit antennas of relays $R_1, \ldots, R_L$. More specifically, the signal reaching the receive antenna of R_l is expressed as in (2). In (2), $\mathbf{h}_{l',l}[t]$ is the $M \times 1$ channel formed between the transmit antennas of Relay $R_{l'}$ and the receive antenna of R_l. The energy that R_l can harvest from $y_{r,l,2}[t]$ is then expressed as [2]:

$$E_{l,2}[t] = \frac{\eta T}{2} \left| \sqrt{P_s} h_{sr,l}[t] + \sum_{l'=1}^{L} \alpha_{l'}[t] \mathbf{w}_{l'}^H[t] \mathbf{h}_{l',l}[t] \right|^2 \tag{5}$$

where $0 \leq \eta \leq 1$ is the efficiency of energy harvesting. The result in (5) is achieved if $x_{s,2} = x_{s,1}$ [2]. Accounting for the ability of R_l to harvest the amount of energy $E_{l,2}[t]$ during the second phase of the t-th time slot, in this work we focus on introducing techniques that exploit this energy in order to provide sustainable relaying techniques, that do not require the wireless powered relays to consume energy resources other than the energy resources that they can harvest. To this end, we introduce the following two energy consumption constraints in our system design:

1) Energy preservation constraint: We impose the constraint that the energy spent by the relay for its transmission during phase II of the t-th time slot, is bounded by the amount of energy that it can harvest during the same time period, i.e., it holds that:

$$\frac{P_{r,l}[t] T}{2} \leq E_{l,2}[t]. \tag{6}$$

2) Energy causality constraint: We further impose the constraint that for each time slot the energy used for beamforming is at most equal to the amount of energy $E_{tot,l}[t-1]$ available to the relay at the end of the previous time slot, i.e. it holds that

$$\frac{P_{r,l}[t] T}{2} \leq E_{tot,l}[t-1]. \tag{7}$$

Clearly, such an assumption presumes that all relays have an amount of energy $E_{tot,l}[0] = E_{0,l} > 0$ at the beginning of transmission, such as to support the transmission that takes place during the first time slot.

By introducing a system design that respects the two abovementioned constraints it is easy to see that for each time instance, all relays $R_l, l = 1, \ldots, L$, operate in a manner that ensures that at the end of the t-th slot, the energy resources at $R_l, l = 1, \ldots, L$, are not less than the energy resources available to it at the end of time slot $t-1$, i.e., it holds that:

$$E_{tot,l}[t] = E_{tot}[t-1] + E_{l,2}[t] - \frac{P_{r,l}[t] T}{2} \geq E_{tot}[t-1]. \tag{8}$$

As a result, the relays can operate in a sustainable manner without consuming their own energy resources.

Accounting for these two constraints, in this work, we consider the problem of designing the beamformers $\mathbf{w}_l[t], l = 1, \ldots, L$, such as to maximize the instantaneous rate of the system while respecting the two constraints. The instantaneous rate of this system, measured in $bits/sec/Hz$ is expressed as

$$R[t] = \frac{1}{2} \log_2 (1 + \gamma_{d,2}[t]) \tag{9}$$

$$y_{d,2}[t] = \sum_{l=1}^{L} \alpha_l[t] \mathbf{w}_l^H[t] \mathbf{h}_{rd,l}[t] x_{s,1}[t] + \sum_{l=1}^{L} \beta_l[t] \mathbf{w}_l^H[t] \mathbf{h}_{rd,l}[t] n_{r,l,1}[t] + n_{d,2}[t], \quad (1)$$

$$y_{r,l,2}[t] = \sqrt{P_s} h_{sr,l}[t] x_{s,2}[t] + \sum_{l'=1}^{L} \frac{y_{r,l',2}[t] \mathbf{w}_{l'}^H[t] \mathbf{h}_{l',l}[t]}{\sqrt{g_{sr,l'}[t] P_s + N_0}} + n_{r,2}[t], \quad (2)$$

$$\text{with } \alpha_l[t] = h_{sr,l}[t] \sqrt{\frac{P_s}{g_{sr,l}[t] P_s + N_0}}, \ \beta_l[t] = \frac{1}{\sqrt{g_{sr,l}[t] P_s + N_0}}, \text{ and } g_{sr,l}[t] = |h_{sr,l}[t]|^2. \quad (3)$$

where $\gamma_{d,2}[t]$ is the SNR for the information signal $y_{d,2}[t]$ in (1). Moreover, the SNR $\gamma_{d,2}[t]$ can be found, after some algebraic manipulations, to be equal to:

$$\gamma_{d,2}[t] = \frac{\left| \sum_{l=1}^{L} \alpha_l[t] \mathbf{w}_l^H[t] \mathbf{h}_{rd,l}[t] \right|^2}{N_0 \sum_{l=1}^{L} \beta_l^2[t] \left| \mathbf{w}_l^H[t] \mathbf{h}_{rd,l}[t] \right|^2 + N_0}. \quad (10)$$

As a result, exploiting the monotonic relation between $R[t]$ and $\gamma_{d,2}[t]$, we can solve the rate-maximization problem by equivalently solving the SNR maximization problem. Dropping, for the ease of presentation, time dependency, and introducing the vectors and matrices given in (12) and (13), the SNR in (10) is equivalently written as the objective function in (11). In addition, it is easy to show that the constraints presented in (11) are essentially the energy preservation and energy causality constraints given in (6) and (7). Hence, optimization problem (11) describes the SNR maximization problem. Having formulated the SNR maximization problem, in the following section we present our approach for solving this optimization problem.

III. The problem of rate-optimal cooperative beamforming

In order to solve the optimization problem in (11), we start by introducing the vectors

$$\begin{aligned} \mathbf{z} &= \left[\Re\{\mathbf{w}\}^T, \Im\{\mathbf{w}\}^T \right]^T, \\ \text{and} \quad \mathbf{b}(b, \mathbf{h}) &= \left[\Re\{b^* \mathbf{h}\}^T, \Im\{b^* \mathbf{h}\}^T \right]^T \end{aligned} \quad (14)$$

and parametric matrices:

$$\boldsymbol{\mathcal{F}}(\mathbf{V}) = \begin{bmatrix} \Re\{\mathbf{V}\} & -\Im\{\mathbf{V}\} \\ \Im\{\mathbf{V}\} & \Re\{\mathbf{V}^H\}, \end{bmatrix}, \boldsymbol{\mathcal{A}}(\mathbf{V}) = \frac{\boldsymbol{\mathcal{F}}(\mathbf{V}) + \boldsymbol{\mathcal{F}}(\mathbf{V})^T}{2}, \quad (15)$$

such as to rewrite (11) as:

$$\begin{aligned} \text{minimize:} \quad & \frac{\mathbf{z}^T \mathbf{A}_1 \mathbf{z} + 2\mathbf{b}_1^T \mathbf{z} + c_1}{\mathbf{z}^T \mathbf{A}_2 \mathbf{z} + 2\mathbf{b}_2^T \mathbf{z} + c_2} \\ \text{subject to:} \quad & \mathbf{z}^T \mathbf{E}_l \mathbf{z} + 2\mathbf{e}_l^T \mathbf{z} + e_l \leq 0, l = 1, \ldots, L, \\ & \mathbf{z}^T \mathbf{F}_l \mathbf{z} + 2\mathbf{f}_l^T \mathbf{z} + f_l \leq 0, l = 1, \ldots, L, \end{aligned} \quad (16)$$

where

$$\begin{aligned} &\mathbf{A}_1 = -\boldsymbol{\mathcal{A}}\left(\mathbf{h}_{rd}\mathbf{h}_{rd}^H\right), \mathbf{A}_2 = \boldsymbol{\mathcal{A}}(\mathbf{C}), \\ &\mathbf{E}_l = \boldsymbol{\mathcal{A}}\left(\mathbf{D}_l - \eta \mathbf{h}_l \mathbf{h}_l^H\right), l = 1, \ldots, L, \\ &\mathbf{F}_l = \boldsymbol{\mathcal{A}}(\mathbf{D}_l), l = 1, \ldots, L, \\ &\mathbf{b}_1 = \mathbf{0}, \mathbf{b}_2 = \mathbf{0}, \\ &\mathbf{e}_l = -\eta \mathbf{b}\left(\sqrt{P_s} h_{sr,l}, \mathbf{h}_l\right), l = 1, \ldots, L, \\ &\mathbf{f}_l = \mathbf{0}, l = 1, \ldots, L, \\ &c_1 = 0, c_2 = 1, \\ &e_l = -\eta |h_{sr,l}|^2 P_s, l = 1, \ldots, L, \\ &f_l = -b_l. \end{aligned} \quad (17)$$

One can recognize optimization problem (16) as a Quadratically Constrained Quadratic Ratio (QCQR) minimization problem [9]. While obtaining an exact solution for such problems in not possible, suboptimal solutions can be obtained by reducing the problem to a Quadratically Constrained Quadratic Programming (QCQP) problem. In Appendix A, the procedure for obtaining such solutions is presented.

A. Complexity considerations

As it can be seen in Appendix A, the problem of optimal cooperative beamforming reduces to a QCQP problem in a $2ML+1$-dimensional space, in the presence of $2L+1$ constraints. The computational complexity of solving such a problem is related to the method that is adopted for solving the associated relaxed semidefinite programming problem. Assuming that the interior point method is used for this purpose, this results in a worst case complexity of [10]:

$$\mathcal{O}\left((2ML+1)^4 \sqrt{2ML+1} \log(1/\epsilon) \right), \quad (18)$$

where $\epsilon \geq 0$ is a predetermined solution accuracy.

IV. Numerical results and discussion

We now consider the application of the proposed beamforming scheme. We study a system employing two relays, each equipped with three antennas with $M = 2$ of them being used in order to transmit data to D. We set S as the origin of our coordinate system, and select the location of D to be at coordinates $(d_{sd} = 50m, 0)$, as in [6], where d_{sd} is the $S \to D$ distance. For the relay positions, we set the position of relay R_1 at coordinates $(d_{sd}/2, d_{sd}/4)$ and consider different placements of the form $(d_{sd}/2, -y)$ for relay R_2. Clearly, as

$$\begin{aligned} &\text{maximize}: \quad \gamma_{d,2} = \frac{1}{N_0} \frac{\mathbf{w}^H \mathbf{h}_{rd} \mathbf{h}_{rd}^H \mathbf{w}}{\mathbf{w}^H \mathbf{C} \mathbf{w} + 1} \\ &\text{subject to:} \quad \underbrace{\mathbf{w}^H \left(\mathbf{D}_l - \eta \mathbf{h}_l \mathbf{h}_l^H \right) \mathbf{w} - 2\eta \Re \left\{ h_{sr,l}^* \sqrt{P_s} \mathbf{w}^H \mathbf{h}_l \right\} - \eta P_s \left| h_{sr,l} \right|^2 \leq 0}_{\text{Energy preservation constraint}}, \quad \underbrace{\mathbf{w}^H \mathbf{D}_l \mathbf{w} \leq b_l = \frac{2E_{tot,l}\left[t-1\right]}{T}}_{\text{Energy causality constraint}} \quad l = 1, \ldots, L, \end{aligned} \tag{11}$$

where for ease of presentation we have dropped time dependency and we have defined the augmented vectors

$$\mathbf{w} = \left[\mathbf{w}_1^T, \ldots, \mathbf{w}_L^T\right]^T, \quad \mathbf{h}_{rd} = \left[\alpha_1 \mathbf{h}_{rd,1}^T, \ldots, \alpha_L \mathbf{h}_{rd,L}^T\right]^T, \quad \mathbf{h}_l = \left[\alpha_1 \mathbf{h}_{1,l}^T, \ldots, \alpha_L \mathbf{h}_{L,l}^T\right]^T \tag{12}$$

and the matrices

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2 & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{C}_L \end{bmatrix}, \quad \mathbf{D}_l = \begin{bmatrix} \mathbf{D}_{l,1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{l,2} & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{D}_{l,L} \end{bmatrix}, \quad \text{where} \quad \mathbf{C}_l = \beta_l^2 \mathbf{h}_{rd,l} \mathbf{h}_{rd,l}^H \quad \text{and} \quad \mathbf{D}_{l,k} = \begin{cases} \mathbf{0}, & \text{if } l \neq k \\ \mathbf{I}, & \text{if } l = k. \end{cases} \tag{13}$$

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y increases, the pathloss on both the $S \to R_2$ and $R_2 \to D$ links also increases, limiting the benefits that the additional relay can deliver.

To consider wireless channel effects, we adopt an exponential pathloss model, and use Rayleigh fading to model multipath. We therefore model $S \to R_l$ and $R_l \to D$, $l = 1, 2$, as follows:

$$h_{sr,l} = \frac{1}{\sqrt{d_{sr,l}^m}} \tilde{h}_{sr}, \ \mathbf{h}_{rd,l} = \frac{1}{\sqrt{d_{rd,l}^m}} \tilde{\mathbf{h}}_{rd,l}, \tag{19}$$

where $d_{sr,l}$ is the distance between S and R_l, and $d_{rd,l}$ is the distance between R_l and D. In (19), due to Rayleigh fading, it holds that $\tilde{h}_{s,lr} \sim \mathcal{CN}(0,1)$, and $\tilde{\mathbf{h}}_{rd,l} \sim \mathcal{CN}(\mathbf{0}_M, \mathbf{I}_M)$. Moreover, we set $m = 2.7$, as in [11]. Concerning the time-characteristics of the communications channels, we have modeled communications channels as autoregressive processes, i.e., it holds that:

$$\begin{aligned} \tilde{h}_{sr,l}[t] &= \sqrt{\theta} \tilde{h}_{sr,l}[t-1] + \sqrt{1-\theta} \hat{h}_{sr,l}[t], \\ \tilde{\mathbf{h}}_{rd,l}[t] &= \sqrt{\theta} \tilde{\mathbf{h}}_{rd,l}[t-1] + \sqrt{1-\theta} \hat{\mathbf{h}}_{rd,l}[t], \end{aligned} \tag{20}$$

where $\hat{h}_{sr,l} \sim \mathcal{CN}(0,1)$, $\hat{\mathbf{h}}_{rd,l} \sim \mathcal{CN}(\mathbf{0}_M, \mathbf{I}_M)$. For the sake of simplicity, we select the same value $\theta = 0.75$ for all channel processes. Finally, we set the energy recycling channel to be equal to $\mathbf{h}_{l,l} = \sqrt{\beta_l}\,[1,1]^T$, where $\beta_l = -15dB$, as in [2], and the efficiency of energy harvesting to be equal to $\eta = 0.8$.

With these assumptions, in Fig. 1 we present the average achievable rate, calculated by averaging over a sequence of 10^3 time slots, as a function of the average SNR at the $S \to R_1, l = 1, 2$, link, defined as

$$\bar{\gamma}_{SR,1} = \frac{P_s}{d_{sr,1}^m N_0}, \tag{21}$$

where $d_{sr,1}$ is the distance between S and R_1. The channel realizations corresponding to these time slots were generated based on model (20). For the sake of comparison with existing schemes, in Fig. 1 we also present the achievable rate for a modified version of the single relay scheme presented in [2] (obtained by introducing the causality constraint presented in (7) in the system design), and assuming that only relay R_1 is present. For both the single and multi-relay cooperative beamforming schemes, we set $E_{tot,1}[0] = E_0$ with E_0 selected such that $E_0/N_0 = 30dB$. Moreover, for the two-relay scheme, we also set $E_{tot,2}[0] = E_0$. Based on the results of Fig. 1, we observe that our system design employing two relays, significantly outperforms the single relay system, even for large values of y, i.e., even for placements of relay R_2 that result in significantly increased losses on the $S \to R_2 \to D$ link, as compared to the $S \to R_1 \to D$ link. In more detail, even for $y = 3d_{sd}/4$ that corresponds to $5dB$ of additional pathloss on the $S \to R_2$ and $R_2 \to D$ links (as compared to the pathloss experienced on the $S \to R_1$ and $R_1 \to D$ links), for $\bar{\gamma}_{SR,1}$ SNR values higher than $30dB$, the use of the cooperative beamforming scheme results in a performance improvement of approximately 25% as compared to the single relay scheme. More importantly, for better placements of relay R_2, the performance improvement is even more substantial. For example, with a symmetric positioning of the relays, i.e., setting $y = d_{sd}/4$, for $\bar{\gamma}_{SR,1}$ values that are higher than $30dB$, the performance gain for the two-relay scheme is higher than 100% as compared to the single relay scheme, and the two-relay scheme results in approximately doubling the achievable average rate, as shown in Fig. 1.

Finally, we now consider the effects of increasing the number of antennas at the relays. To this end, in Fig. 2 we present the achievable average rate, again with averaging performed over 10^3 consecutive time slots, for a system with $M = 2$ and $M = 3$ transmit antennas, as a function of the SNR $\bar{\gamma}_{SR,1}$. For the case that $M = 3$, we have set the channel $\mathbf{h}_{l,l}$ to be equal to $\mathbf{h}_{l,l} = \sqrt{\beta_l}\,[1,1,1]^T$ with $\beta_l = -15dB$, similar to the two-antenna case. For the simulations presented in Fig. 2 we have considered the symmetric case where relays R_1 and R_2 are placed at coordinates $(d_{sd}/2, d_{sd}/4)$ and $(d_{sd,2}, -d_{sd}/4)$. The results shown in Fig. 2 indicate that the performance benefits that were observed for the case of $M = 2$ are also evident in the case that $M = 3$, with the presence of the additional relay resulting in performance gains that are equal or even higher than 100%, for $\bar{\gamma}_{SR,1}$ values that are higher than $30dB$

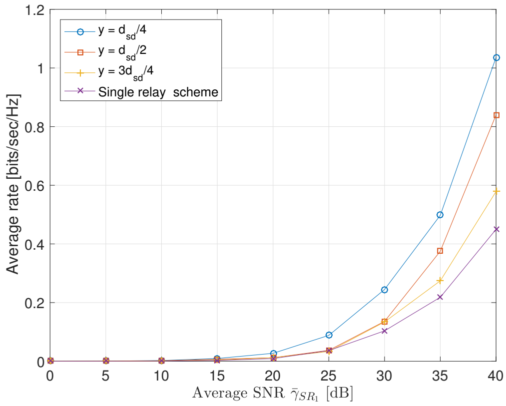

Fig. 1. The achievable rate as a function of the $S \rightarrow R_1$ for different positions of the relay R_2.

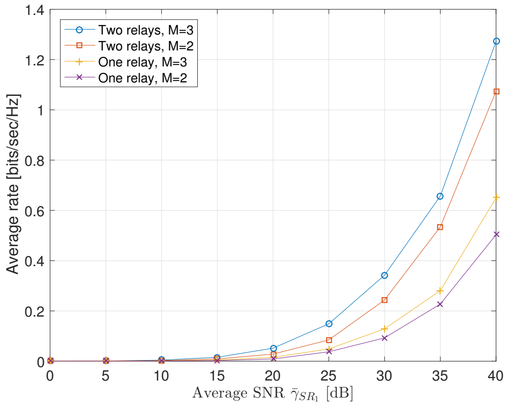

Fig. 2. The achievable performance for single and multi-relay schemes as a function of the $S \rightarrow R_1$ SNR for different number of transmit antennas at the Relays.

for the multi-relay scheme with $M = 3$, as compared to the single relay scheme. Moreover the performance improvement that additional antennas can deliver, as seen in Fig. 2, are significant. For example, focusing on the high SNR regime, (i.e., for $\bar{\gamma}_{SR,1} \geq 30dB$) the use of 3 transmit antennas on the relays of the two-relay scheme, results in a performance improvement, as compared to the two-relay scheme with two transmit antennas at the relay, that is, in the worst case, higher than 20%.

V. Conclusion

We have proposed a new scheme for exploiting multiple relays in energy harvesting/energy recycling assisted cooperative communications. For these schemes, we have studied the problem of beamformer design, presenting algorithmic solutions. By means of simulations, we have verified the significant gains that the proposed multiple relay scheme can deliver, as compared to single relay schemes.

Appendix A
Solving the Cooperative Beamforming Problem

In order to solve optimization problem (16), let us start by introducing an auxillary variable u and define the vector $\tilde{\mathbf{z}}$ as $\tilde{\mathbf{z}} = u\mathbf{z}$. Optimization problem (16) then becomes equivalent to problem [9]:

$$\begin{aligned} \text{minimize: } & \tilde{\mathbf{z}}^T\mathbf{A}_1\tilde{\mathbf{z}} + 2\mathbf{b}_1^T\tilde{\mathbf{z}} + c_1u^2 \\ \text{subject to: } & \tilde{\mathbf{z}}^T\mathbf{A}_2\tilde{\mathbf{z}} + 2\mathbf{b}_2^T\tilde{\mathbf{z}} + c_2u^2 = 1 \\ & \tilde{\mathbf{z}}^T\mathbf{E}_l\tilde{\mathbf{z}} + 2\mathbf{e}_l^T\tilde{\mathbf{z}} + e_lu^2 \leq 0, l = 1,\ldots,L, \\ & \tilde{\mathbf{z}}^T\mathbf{F}_l\tilde{\mathbf{z}} + 2\mathbf{f}_l^T\tilde{\mathbf{z}} + f_lu^2 \leq 0, l = 1,\ldots,L, \end{aligned} \tag{22}$$

provided that the optimal solution to (22) does not correspond to $u = 0$. In order to prove that the solution to (22) does not correspond to $u = 0$, let us start by considering a feasible point of the form $\{\tilde{\mathbf{z}}, 0\}$ for optimization problem (22). Accounting for feasibility, this practically means that vector $\tilde{\mathbf{z}}$ simultaneously satisfies the energy causality constraints

$$\tilde{\mathbf{z}}^T\mathbf{F}_l\tilde{\mathbf{z}} \leq 0, \quad \forall l = 1,\ldots,L. \tag{23}$$

Equivalently, by defining the vector $\tilde{\mathbf{w}}$ as

$$\tilde{\mathbf{w}} = \tilde{\mathbf{z}}\,(1 : ML) + j\tilde{\mathbf{z}}\,(ML + 1 : 2ML)\,, \tag{24}$$

this means tht vector $\tilde{\mathbf{w}}$ simultaneously satisfies the constraints

$$\tilde{\mathbf{w}}^H\mathbf{D}_l\tilde{\mathbf{w}} \leq 0, \quad \forall l = 1,\ldots,L. \tag{25}$$

Nevertheless, based on the definition of matrices $\mathbf{D}_l$ this can only be satisfied provides that $\tilde{\mathbf{w}} = \mathbf{0}_{LM}$. Hence, the only feasible solution of (22) for which $u = 0$ is the all zero vector. On the other hand, it is easy to show that any feasible solution that does not correspond to $\tilde{\mathbf{z}} = 0$ achieves a lower value for the objective value of (22), since $\mathbf{A}_1$ is negative definite, while in addition we have that $\mathbf{a}_1 = 0$ and $a_1 = 0$. As a result, the solution to (22) is not equal to $\{\tilde{\mathbf{z}} = 0, u = 0\}$, provided that at least one feasible beamformer can be found. Clearly such a beamformer is easy to construct. Optimization problem (16) can therefore be solved by applying the optimization algorithm presented in [9], in order to transform it into the following problem:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}}: \; & \mathbf{x}^T\boldsymbol{\mathcal{M}}\left(\mathbf{A}_1, \mathbf{b}_1, c_1\right)\mathbf{x} \\ \text{subject to: } & \mathbf{x}^T\boldsymbol{\mathcal{M}}\left(\mathbf{A}_2, \mathbf{b}_2, c_2\right)\mathbf{x} = 1 \\ & \mathbf{x}^T\boldsymbol{\mathcal{M}}\left(\mathbf{E}_l, \mathbf{e}_l, e_l\right)\mathbf{x} \leq 0, \; l = 1,\ldots,L, \\ & \mathbf{x}^T\boldsymbol{\mathcal{M}}\left(\mathbf{F}_l, \mathbf{f}_l, f_l\right)\mathbf{x} \leq 0, \; l = 1,\ldots,L, \end{aligned} \tag{26}$$

where

$$\boldsymbol{\mathcal{M}}\left(\mathbf{A}, \mathbf{b}, c\right) = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{b}^T & c \end{bmatrix}. \tag{27}$$

One can easily recognize that problem (26) is a Quadratically Constrained Quadratic Programming (QCQP) problem. To solve this problem, we start by defining matrix $\mathbf{X} = \mathbf{x}\mathbf{x}^T$, and using the cyclic property of trace. We can then show that for any symmetric matrix $\boldsymbol{\mathcal{M}}$ it holds that

$$\mathbf{x}^T\boldsymbol{\mathcal{M}}\mathbf{x} = \mathrm{Tr}\left(\boldsymbol{\mathcal{M}}\mathbf{X}\right), \tag{28}$$

and express (26) as:

$$\begin{aligned} &\underset{\mathbf{X}}{\text{minimize}}: \text{Tr}\left(\boldsymbol{\mathcal{M}}\left(\mathbf{A}_1, \mathbf{b}_1, c_1\right)\mathbf{X}\right) \\ &\text{subject to: } \text{Tr}\left(\boldsymbol{\mathcal{M}}\left(\mathbf{A}_2, \mathbf{b}_2, c_2\right)\mathbf{X}\right) = 1 \\ &\qquad \text{Tr}\left(\boldsymbol{\mathcal{M}}\left(\mathbf{E}_l, \mathbf{e}_l, e_l\right)\mathbf{X}\right) \leq 0, \quad l = 1, \ldots, L \\ &\qquad \text{Tr}\left(\boldsymbol{\mathcal{M}}\left(\mathbf{F}_l, \mathbf{f}_l, f_l\right)\mathbf{X}\right) \leq 0, \quad l = 1, \ldots, L \\ &\qquad \mathbf{X} \succeq \mathbf{0}, \text{rank}\left(\mathbf{X}\right) = 1. \end{aligned} \tag{29}$$

Optimization problem (29) is a non-convex optimization problem, due to the rank-one constraint [10]. Nevertheless, the relaxed problem resulting by dropping the rank-one constraint can be solved using standard convex optimization techniques [10]. While a solution to this relaxed problem can be obtained, in general, there is no guarantee that this solution satisfies the rank-one constraint of problem (29), hence there is no guarantee that the solution to the relaxed problem is a solution to (29). Nonetheless, suboptimal solutions to (29) can be constructed based on the solution of its relaxed version. Such a solution can be obtained if $\mathbf{X}$ is chosen as

$$\mathbf{X} = \lambda_1 \mathbf{e}_1 \mathbf{e}_1^T \tag{30}$$

where λ_1 is the largest eigenvalue of $\mathbf{X}^*$ and $\mathbf{e}_1$ the eigenvector corresponding to this eigenvalue. As an alternative to this process for obtaining rank-one solution, the randomization process in [10] can be applied. Nevertheless, for the sake of simplicity in this work we use the expression in (30). The problem of approximating the solution to problem (16) is then summarized in Algorithm 1.

Algorithm 1 Algorithm for solving optimization problem (16)

Input: $\mathbf{A}_1, \mathbf{A}_2, \mathbf{E}_l, \mathbf{F}_l, \mathbf{a}_1, \mathbf{a}_2, \mathbf{e}_l, \mathbf{f}_l, a_1, a_2, e_l, f_l,$
$l = 1, \ldots, L$

Output: Optimal beamformer $\mathbf{z}_{opt}$

1: Solve the optimization problem obtained by relaxing, i.e. omitting, the rank-one constraint in (29) to obtain a solution $\mathbf{X}_{opt}$.

2: Find the eigenvector $\mathbf{e}_1 = \left[\tilde{\mathbf{z}}_0^T, u\right]$ that corresponds to the maximum eigenvalue of $\mathbf{X}_{opt}$

3: Calculate $\mathbf{z}_{opt}$ as

$$\mathbf{z}_{opt} = \frac{\tilde{\mathbf{z}}_0}{u} \tag{31}$$

Acknowledgment

This publication has emanated from research conducted with the financial support of Science Foundation Ireland (SFI) under Grant Number 13/RC/2077.

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